Nonlinear Dyn (2021) 104:4117–4147
https://doi.org/10.1007/s11071-021-06535-8

ORIGINAL PAPER

To keep faith with homoskedasticity or to go back to heteroskedasticity? The case of FATANG stocks

José Dias Curto

Received: 9 February 2021 / Accepted: 10 May 2021 / Published online: 28 May 2021

Abstract Did the pattern of US stock market volatility change due to COVID-19 or have the US stock markets been less volatile despite the pandemic shock? And as for tech stocks, are they even less volatile than the market overall? In this paper, we provide evidence in favor of a "quietness" in the stock markets, interrupted by COVID-19, by analyzing dispersion, skewness and kurtosis characteristics of the empirical distribution of nine returns series that include individual FATANG stocks (FAANG: Facebook, Amazon, Apple, Netflix and Google; plus Tesla) and US indices (S&P 500, DJIA and NASDAQ). In comparison with the years before, the daily average return after COVID-19 was 6.48, 2.58 and 2.34 times higher for Tesla, Apple and NASDAQ, respectively. In terms of volatility, the increase was more pronounced in the three stock indices when compared to the individual FATANG stocks. This paper also puts forward a new methodology based on semi-variance and semi-kurtosis. While the value of the ratio between semi-kurtosis and kurtosis is always higher than 70% for the three US stock indices, in the case of stocks the opposite is true, which highlights the importance of large positive returns when compared to negative ones. Structural breaks and conditional heteroskedasticity are also analyzed by considering the traditional symmetrical and asymmetrical GARCH models. We show that in the most recent past, despite the COVID-19 pandemic, the FATANG tech stocks are characterized mostly by conditional homoskedasticity, while the returns of US stock indices are characterized mainly by conditional heteroskedasticity.

Keywords US stock markets · Volatility · Persistence · Half-life · Semi-kurtosis

This work was supported by Fundação para a Ciência e a Tecnologia, Grant UIDB/00315/2020.

J. D. Curto (✉)
Instituto Universitário de Lisboa (ISCTE-IUL), BRU-UNIDE, Lisboa, Portugal
e-mail: dias.curto@iscte-iul.pt

Department of Quantitative Methods for Management and Economics, BRU-UNIDE, Lisboa, Portugal

1 Introduction

Since the 2008 subprime mortgage crisis, the US stocks and indices have begun a remarkable recovery, giving rise to the longest bull market in the history of the US financial markets. As can be seen in Fig. 1 the S&P 500[1] achieved its minimum (676.53) in March 9, 2009, and from there it has almost always been going up.

This stunning growth is evident from the numbers in Table 1. The stock price of Netflix, for example, grew 9617.3% in 11 years giving rise to a remarkable 51.6% geometric average annual return. High growth rates also characterize other US stocks and indices.

[1] US indices: Standard & Poor's 500 (S&P 500), Dow Jones Industrial Average (DJIA) and NASDAQ Composite.

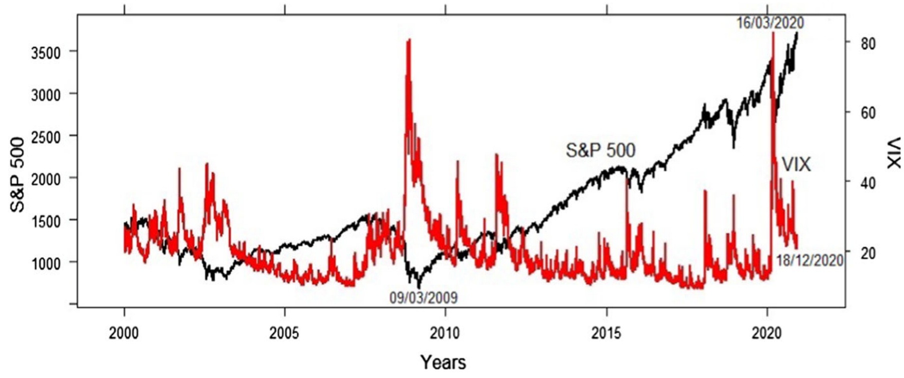

Fig. 1 S&P 500 and VIX

Table 1 Growth of US stocks and indices

Index/stock	Prices		Growth	
	09/03/2009	18/12/2020	Total (%)	Average (%)
S&P 500	676.53	3709.41	448.3	16.7
DJIA	6547.05	30179.05	361.0	14.9
NASDAQ	1268.64	12755.64	905.5	23.3
AMAZON	60.49	3201.65	5192.9	43.4
APPLE	2.56	126.66	4847.7	42.6
NETFLIX	5.50	534.45	9617.3	51.6
GOOGLE	144.50	1731.01	1097.9	25.3

And with regard to volatility, what has been happening in the most recent past? Before 2020, there was a "quietness" in the stock markets occasionally interrupted by short stressful periods of increasing volatility. However, the market quickly recovered and returned to its upward trend. The Fed's predictable policy is one reason experts give as to why stock-market volatility has been historically low. And, indeed, over the past few years, Fed policy has been cautious and relatively predictable.

In 2018 and 2019, with the US economy humming along and interest rates no longer so close to zero, the case for strong forward guidance regarding future policy actions was becoming less compelling. According to John Williams, in his first speech as President of the New York Fed, it was not so clear as before whether interest rates should go up or down; and explicit forward guidance, or promises about how the Fed will behave, would no longer be appropriate. Thus, with the future direction of policy apparently being less clear than during the previous few years, there would be more uncertainty and volatility in the stock markets,[2]. Despite this, however, US stocks and indices had reached new historical highs by the end of 2019.

The year 2020, particularly March of that year, was characterized by unusual variations in stock prices, which led to a period of extremely high volatility. This was due, however, to the spread of COVID-19 worldwide,[3] and not because of a Fed policy change. As can be seen in Fig. 1, the Chicago Board Options Exchange (CBOE) Volatility Index (VIX), a popular measure of the stock market's expectation of volatility based on S&P 500 index options, achieved its 82.69 highest value in March 16. Bearing in mind that VIX values greater than 30 are commonly associated with high instability, values below 20 clearly correlate to less distressing times in the financial markets.

After the intervention of Governments and Central Banks, with stimuli of billions of dollars and euros, the capital markets calmed down after March, 2020

[2] Greg Robb at https://www.marketwatch.com/ September 28, 2018, for details.

[3] Consequences of COVID-19 in terms of risk and uncertainty are well documented in [43] due to the increase in the number of hospitalizations, morbidity or over mortality.

(the VIX quickly returned to pre-COVID levels), which poses another million dollar question for financial analysts and investors: are these recent stable market conditions able to continue or can a new stressed regime of volatility be expected to arise in the near future? Could the severity and frequency of the swings seen on Wall Street last year (the markets have not seen moves this wild since the subprime mortgage financial crisis) be here to stay or are they just a brief period of turmoil and the calm will return soon with the economic recovery? Thus, the question "To keep faith with homoskedasticity or to go back to heteroskedasticity?" would seem to make sense.

As noted early in the seminal papers of [29] and [15], financial time series vary systematically with time, showing periods of enormous unpredictability and volatility, followed by times of low instability. Regardless of these early investigations, and the importance of modelling and forecasting volatility in financial markets (see, for instance, [30]), efforts to model volatility dynamics were only developed in the later decades of the twentieth century. Up until then, the homoskedasticity of errors had been assumed in traditional econometric models, i.e., the focus of financial time series modelling was the conditional first moment, neglecting any temporal dependencies in the higher-order moments.

The volatility of financial assets has been extensively studied for the last thirty years, since the seminal papers of [13] and [4] were published, and remains a hot topic of investigation due to its importance for investors, financial analysts and academics. Volatility modelling, and especially volatility dynamics, is important for decision-making in financial markets involving derivative prices, bonds, leverage ratios, credit spreads and portfolio decisions. Recently, using data from 50 countries, Xue [53] investigates the effect of financial sector development on growth volatility. The empirical results show that broadly speaking, while the aggregate growth volatility declined from 1997 to 2014, growth volatility in the most developed countries was much smaller than in other countries. [41] investigated the determinants of six euro area sovereign bond yield spreads between June, 2006 and January, 2017, by estimating original Panel-GARCH models, incorporating key stylized features of volatility dynamics, such as extreme persistence, asymmetry [32] and risk premia effects. They found, in accordance with the financial literature, significant and considerable volatility persistence, despite the presence of asymmetric effects on the volatility process being seemingly negligible. Empirical evidence from [42]'s analysis of the spillover effects in interbank money markets shows that money markets are profoundly interrelated, displaying dynamic cross-market impacts. Besides, they highlight the pertinence of conditional covariances, showing that the volatility spillovers are time-varying and strongly related to major economic events, increasing during periods of higher uncertainty. This supports the need to closely monitor the evolution of money markets.

Not so recently, but still focusing on volatility, [36] examined volatility spillovers between oil prices and emerging economies. Their significant results show the weak integration of the Chinese financial markets, energy markets and the US stock market. At the same time, the Brazilian, Indian and Russian markets were found to be more sensitive to international shocks resulting from US markets, and also to the instability of prices in the energy markets, particularly with regard to oil price uncertainty.

Due to changes in volatility, Rapach and Strauss [40] examined the importance of structural breaks in exchange rate volatility by means of in-sample and out-of-sample tests. The results indicate the presence of structural breaks in the unconditional variance of seven out of eight US dollar exchange rate return series over the 1980-2005 period. This would suggest unstable results, with GARCH(1,1) estimates frequently shifting across subsamples as a result of the structural breaks, and particularly impacts volatility persistence over time.

Structural breaks in volatility have also been tested and considered by [8,50,54], among many others. Using daily data for six major international stock market indices and a modified EGARCH specification, Curto et al. [8] analyze the relationship between stock market returns, volatility and trading volume by proposing a new nonlinear conditional variance model with multiple regimes and volume effects (MSV-EGARCH). By using the Harvey–Newbold test for multiple forecast encompassing, they show that the more complex MSV-EGARCH threshold structure dominates the competing standard asymmetric models (GJR and EGARCH) in terms of forecasting ability, for several of the considered stock indices. Smith [50] compares the ability of traditional diagnostic tests to detect various sorts of breaks in GARCH models. The results show that the robust LM tests proposed by

Wooldridge [55] have no power to detect structural breaks in GARCH models. When CUSUM- and LM-based structural break tests are considered, the results point to an excellent size when the data is Gaussian. However, while the CUSUM tests tend to over-reject in fat tails returns, whatever the sample size, the LM-based tests show approximately the correct size, and that the power to detect various sorts of breaks in the dynamics of conditional volatility is very high. Wen et al. [54] analyzed the interaction between oil prices and the US dollar exchange rate. The results indicate that ignoring structural breaks can increase the negative volatility correlation between the oil price and USD exchange rate markets, and have a particular impact during the financial crisis. Shen et al. [48] studied the volatility of the Bitcoin cryptocurrency highlighting the importance of jumps and structural breaks in forecasting its volatility. In the out-of-sample analysis, they found that the HARQ-F-J model is the best one, which shows the importance of the temporal variation and squared jump components at different time spans.

More recently, the COVID-19 pandemic triggered a number of new research papers, in line with ours, aimed at assessing the impact of corona virus on the volatility of financial asset returns. By using an extended GARCH-MIDAS model and a newly developed Infectious Disease Equity Market Volatility Tracker (EMV-ID), Bai et al. [2] investigate the effects of COVID-19 on the volatility of the stock markets of the USA, China, the UK and Japan through January 2005 to April 2020. The empirical results show that, with an up to 24-month lag, the infectious disease pandemic has had significant positive impacts on the permanent volatility, even after controlling for several factors, namely the influence of past realized volatility, global economic policy uncertainty and the volatility leverage effect. Salisu and Vo [45] assessed the relevance of health-news trends in the predictability of stock returns. The results show that the model incorporating the health-news index outperforms the benchmark historical average model, highlighting the importance of health news as a good predictor (due to its significance) of stock returns since the emergence of the pandemic. Sadefo et al. [47] found, based on the Asymmetric Power GARCH model, that COVID-19 is having a substantial negative impact and reduces the US and Japan's market returns. Moreover, the influence of COVID-19 on the stock market variance of the USA, Germany, and Italy is greater in comparison with the Global Financial Crisis of 2008.

This small selection of papers illustrates how the volatility of financial asset returns remains a very important investigation topic in finance. This being so, and due to the lack of empirical analyses involving FATANG stocks, the motivation of this study is to address (in accordance with the expected structural breaks in volatility) how the three main stylized facts of returns volatility [7]: clustering-ARCH effect, persistence, and asymmetry of the FATANG stocks, have evolved (and changed) over time when compared to the stock markets in general, represented by the S&P 500, NASDAQ and DJIA US indices (see Sect. 3 for a brief description of data). We focus mainly on before and after the COVID-19 shock in 2020.

The contribution is fivefold, theoretical and empirical. First, we expand the acronym FAANG to FATANG by including the Tesla stock. To the best of our knowledge, this is the first time that this new acronym is used.

Second, a new methodology based on semi-variance and semi-kurtosis (Sect. 2) is proposed and described to compare the two sides of the returns mean and to distinguish between risk and uncertainty; a new indicator based on semi-kurtosis is proposed to evaluate the downside risk. Traditional measures (value-at-risk, for example) only consider the most extreme observations in the left tail of the empirical distribution, disregarding most of the negative variations when assessing risk. Thus, by comparing the two sides of the returns mean (which is close to zero), we take into account all the negative returns, thereby providing a simple but more informative measure of risk. When compared to the standard deviation, the new measure is also more suitable in case of asymmetrical distributions (a common characteristic of financial data). This is our main theoretical contribution. See Sect. 4 for empirical results.

Third, there are many studies that do not consider the effects of structural breaks in volatility on GARCH estimation. Thus, before estimating the GARCH, GJR and EGARCH models, we test for the existence of structural breaks in the unconditional variance of returns of the nine series under analysis. The novelty here resides in the use of the PELT algorithm (which is not so common in finance) to detect the structural breaks. Although the results are similar to those resulting from the traditional modified version of the iterated cumulative sum of squares (ICSS) algorithm, the advantage of PELT is that it can be used directly through the `changepoint` R package.

Fourth, as we expect that some characteristics of the empirical distributions will have evolved (and changed) over time, especially after the United States subprime mortgage crisis and the COVID-19 pandemic, and to notice the temporal changes, all the measures were computed over a rolling window encompassing the previous year of daily observations ($T = 250$). This dynamic form of analysis is also new compared to most other empirical studies (see Sect. 4 for results).

Finally, the empirical results are surprising and point to a "quietness" in the US stock markets that was only momentarily interrupted by COVID-19. The conditional heteroskedasticity is more evident for stock indices, with the ARCH effect having declined in the most recent past (the exception being the year 2020), especially in the case of FATANG stocks (see Sect. 5 for results and discussion).

2 The methods

To answer the question "To keep faith with homoskedasticity or to go back to heteroskedasticity?" we started by computing descriptive statistics (mean, standard deviation, skewness and kurtosis) over a rolling window encompassing the previous year of daily observations ($T = 250$), to notice the temporal changes. ANOVA, Kruskal–Wallis and Levene tests are used to compare means, distributions and variances of returns between different periods. A simple linear regression was also estimated in order to reach a conclusion regarding the linear relationship between the means and variances of returns (see Sects. 4.1 and 4.2).

A new methodology based on semi-variance and semi-kurtosis was proposed, and is described below (see Sect. 2.1), to compare the two sides of the mean and to distinguish between risk and uncertainty. The empirical results appear in Sect. 4.2.

Existent econometric methodologies are also used. To draw conclusions with regard to the autocorrelation of returns, absolute returns and square returns over time, the Ljung–Box (LB) test for up to the tenth-order serial correlation was computed over the rolling window process. The Lagrange multiplier test [13] was used to formally test the presence of conditional heteroskedasticity and the evidence of ARCH effects. An ARMA(4,0) model was estimated, primarily to prefilter the data from linear dependency. See Sect. 4.3.

The econometrical approach is detailed next in Sect. 2.2, and empirical results appear in Sect. 5.

2.1 Semi-variance and semi-kurtosis

In Sect. 4.2, we show that when the individual FATANG stocks are considered, the estimates of skewness can be either positive or negative and the peaks of kurtosis are also explained by large positive returns leading to the positive spikes of the coefficient of skewness. Thus, large positive returns also inflate the kurtosis in the case of FATANG stocks. This empirical evidence has important consequences in terms of risk analysis. Variance (as a measure of volatility) and kurtosis increase mainly due to large negative returns (indices) and also to large positive returns (stocks). However, only the former represent an increase in risk, because the chance of selling the asset at a lower price (below its purchase price) is higher. If an asset receives a large positive return, the event is considered an increase in uncertainty but not in risk. Both large negative and positive returns produce an increase in variance and kurtosis because while they equally take into account movements in either tail of the distribution, only those on the left side are undesirable [19]. Thus, it is important to separate the "bad" volatility and kurtosis from the "good", as the former represents risk, while the latter only represents uncertainty.

Let $\sigma^{2-} = \sum_{t=1}^{T_1} (r_t - \mu)^{2-}$ and $K^- = \sum_{t=1}^{T_1} (r_t - \mu)^{4-}$ represent the downside of the variance and the coefficient of kurtosis, respectively: all the observations that fall below the mean. If we divide σ^{2-} and K^- by T and $T\sigma^4$, respectively, we get the semi-variance [31] and the semi-kurtosis (see [44] for an application). We can also define the upside as $\sigma^{2+} = \sum_{t=1}^{T_2} (r_t - \mu)^{2+}$ and $K^+ = \sum_{t=1}^{T_2} (r_t - \mu)^{4+}$, including all the observations that fall above the mean. With,

$$\sum_{t=1}^{T_1} (r_t - \mu)^{2-} + \sum_{t=1}^{T_2} (r_t - \mu)^{2+} = \sum_{t=1}^{T} (r_t - \mu)^2 ,$$

$$\sum_{t=1}^{T_1} (r_t - \mu)^{4-} + \sum_{t=1}^{T_2} (r_t - \mu)^{4+}$$

$$= \sum_{t=1}^{T} (r_t - \mu)^4 , \quad \text{with } T = T_1 + T_2,$$

obviously being the numerators of the variance and the coefficient of kurtosis, respectively.

In order to evaluate the downside risk, we can divide the semi-variance by the variance and the semi-kurtosis by the kurtosis, respectively, giving rise to the following ratios:

$$R_V = \frac{\sigma^{2-}}{\sigma^2} = \frac{\sum_{t=1}^{T_1} (r_t - \mu)^{2-}}{\sum_{t=1}^{T} (r_t - \mu)^2},$$
$$R_K = \frac{K^-}{K} = \frac{\sum_{t=1}^{T_1} (r_t - \mu)^{4-}}{\sum_{t=1}^{T} (r_t - \mu)^4}, \quad (1)$$

and both measures range between 0 and 1 (the extremes are not included) and the risk decreases when the value of the ratios goes to zero. (For values lower than 0.5, the upside "dominates" the downside.) If the value is 0.5, the two sides are balanced and the distribution is symmetric. The risk increases when the value of the ratios is higher than 0.5. (The downside "dominates" the upside.)

2.2 Econometrical approach

The empirical distribution of a financial asset return can be described as the sum of a predictable part with an unpredictable part:

$$r_t = E\left[r_t \mid \Phi_{t-1}\right] + u_t, \quad (2)$$

where Φ_{t-1} is the relevant information set up to, and including, $t - 1$. For the conditional mean, $E\left[r_t \mid \Phi_{t-1}\right]$, our first intuition was to assume a white noise process, since the empirical distributions of returns under study represent the most liquid and efficient financial markets in the world—as far as equities are concerned—and since this work is primarily dedicated to the dynamics of the variance equation. However, anticipating our findings in the data analysis section, we shall also specify the conditional mean equation as a fourth-order autoregressive process, AR(4), in order to remove the observed linear dependency in returns:

$$r_t = c + \phi_1 r_{t-1} + \phi_2 r_{t-2} + \phi_3 r_{t-3} + \phi_4 r_{t-4} + u_t, \quad (3)$$

where $u_t = z_t \sigma_t$ and the standardized innovations (z_t) are assumed to be independently and identically distributed (i.i.d.) with student's t distribution [5]. This statistical distribution has a long tradition in the econometrics literature as a popular choice for a fat-tailed distribution, since it has a finite second moment (in contrast to stable non-Gaussian distributions), its mathematical properties are well known, it is undemanding to estimate, and is often found capable of capturing the excess of kurtosis observed in financial time-series. Other non-normal alternative distributions have also been used in econometrical literature. Nelson [37] proposed the generalized error distribution (GED), the Laplace distribution has been employed in [18], and [22] used both the student's t and GED as distributional alternative models for innovations. The stable Paretian distributions have also been investigated by [9,27,35].

For the conditional variance of u_t: $E\left[u_t^2 \mid \Phi_{t-1}\right] = \sigma_t^2$, we have considered the most popular conditional heteroskedastic specifications: the symmetric GARCH [4] and the asymmetric GJR [17] and EGARCH [37] models to incorporate the leverage effect (see, for example, [10,51]). As observed by [3], volatility responds asymmetrically to the sign of any change in the price of the financial asset, i.e., volatility increases more after negative changes than after positive changes of the same magnitude. This phenomenon has become known to as the leverage effect (also referred as the Fisher–Black effect).

In this study, via the estimation of asymmetrical models, we want to test whether the leverage effect is also a characteristic of FATANG stocks. We show that in some periods of time, both positive and negative news have the same impact on volatility, and yet have no asymmetrical effect. The leverage effect, however, is still present in all the stock indices analyzed.

Despite the theoretical interest of (p, q) models, the $(1, 1)$ specification is, in general, satisfactory when modeling financial asset returns volatility (see [6] and more recently [20]). Thus, in this paper all conditional heteroskedastic models are of $p = 1, q = 1$ order:

$$\text{GARCH:} \quad \sigma_t^2 = \omega + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \quad (4)$$

$$\text{EGARCH:} \quad \ln \sigma_t^2 = \omega + \alpha_1 \frac{|u_{t-1}|}{\sigma_{t-1}} + \gamma_1 \frac{u_{t-1}}{\sigma_{t-1}} + \beta_1 \ln \sigma_{t-1}^2, \quad (5)$$

$$\text{GJR:} \quad \sigma_t^2 = \omega + \alpha_1 u_{t-1}^2 + \gamma_1 I_{t-1} u_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \quad (6)$$

where $\omega, \alpha_1, \gamma_1$ and β_1 are unknown parameters, $I_{t-1} = 1$ if $u_{t-1} < 0$ and $I_{t-1} = 0$ if $u_{t-1} \geq 0$. The models are estimated through maximum likelihood (MLE). Estimated models are compared based on Bayesian information criteria [49].

Several studies have shown that structural breaks can impact significantly on GARCH model estimation, and

Table 2 Summary statistics of returns

Index/stock	Starting date	# Obs	Mean	Median	Min	Max	St Dev	Skew	Kurt	J–B
S&P 500	29/01/1985	9047	0.033	0.063	– 22.900	10.957	1.163	– 1.252	29.937	0.000
DJIA	30/01/1985	9047	0.035	0.058	– 25.632	10.764	1.148	– 1.596	41.728	0.000
NASDAQ	31/01/1985	9047	0.042	0.113	– 13.149	13.255	1.397	– 0.345	11.809	0.000
FACEBOOK	18/05/2012	2161	0.092	0.106	– 21.024	25.937	2.342	0.335	18.058	0.000
AMAZON	15/05/1997	5939	0.125	0.049	– 28.457	29.618	3.650	0.455	11.941	0.000
TESLA	29/06/2010	2637	0.189	0.116	– 23.652	21.829	3.535	– 0.033	9.039	0.000
APPLE	29/01/1985	9047	0.078	0.009	– 73.125	28.689	2.834	– 1.985	59.105	0.000
NETFLIX	23/05/2002	4677	0.130	0.035	– 52.605	35.223	3.638	– 0.876	26.289	0.000
GOOGLE	19/08/2004	4113	0.086	0.069	– 12.340	18.225	1.915	0.453	12.130	0.000

Skew: Coeff. of skewness, Kurt: Coeff. of Kurtosis and J–B is the p value associated with the Jarque–Bera test

that neglecting structural breaks has important consequences. First, the degree of persistence in the volatility of returns can be overstated [21,28,33]. Secondly, [33,34,38] show that structural breaks can give rise to spurious evidence of long-range dependence or long memory in financial volatility data. Thirdly, in out-of-sample volatility forecasting, the use of an expanding data window (or a fixed data window) is unlikely to perform well in the presence of sudden structural breaks in volatility [40]. Thus, before estimating the GARCH, GJR and EGARCH models, we first tested for the existence of structural breaks in the unconditional variance of returns of the nine series under analysis. These, according to Rapach and Strauss [40], are equivalent to structural breaks in the parameters of the GARCH processes governing the conditional volatility of returns.

To identify structural breaks in the volatility of the nine returns series, we applied a modified version of the [23] iterated cumulative sum of squares (ICSS) algorithm that allows for the dependent processes proposed by Rapach and Strauss [40]. The algorithm is used to test for (potentially multiple) structural breaks in the unconditional variance of daily returns for the nine series under study. For more details about the ICSS algorithm, see also [11,25,26,46], for example. The PELT algorithm [24] was also used, and the results were similar. The advantage of PELT is that it can be used directly through the `changepoint` R package. Thus, the samples are divided into distinct periods.

In this study, testing for the presence of structural breaks in volatility has two main objectives. Firstly, did the COVID-19 pandemic result in a structural break for all the returns series, or just for some? We show that no changes in volatility have occurred for some of the FATANG stocks, which would indicate conditional homoskedasticity. Secondly, we wanted to analyze how the three stylized facts of the returns' volatility [1]: clustering, persistence, and asymmetry have evolved over time, and whether the differences are statistically significant.

3 The data

The data sets we will analyze in this paper are the daily closing prices and the continuously compound returns of the three main US stock indices: S&P 500, DJIA and NASDAQ, and six US stocks.[4] Five of the six, under the acronym FAANG, refer to the stocks of the five most popular and best-performing American technology companies: Facebook, Amazon, Apple, Netflix and Alphabet (formerly known as Google). In this paper we also include Tesla, and have changed the acronym to FATANG. The starting date is not common to all series (see Table 2 for details), but we try to cover as lengthy a period as possible. For example, in the case of FATANG stocks, prices go back to the first day each company was listed on the New York Stock Exchange (except Apple), with the end date being December 18, 2020.

We analyze the continuously compounded percentage rates of return (adjusted for dividends) which are calculated by taking the first differences of the logarithm of the series (P_t is the closing value for each index or stock at time t):

$$r_t = 100 \times \left[\ln\left(P_t\right) - \ln\left(P_{t-1}\right)\right]. \tag{7}$$

[4] Data source is Yahoo Finance.

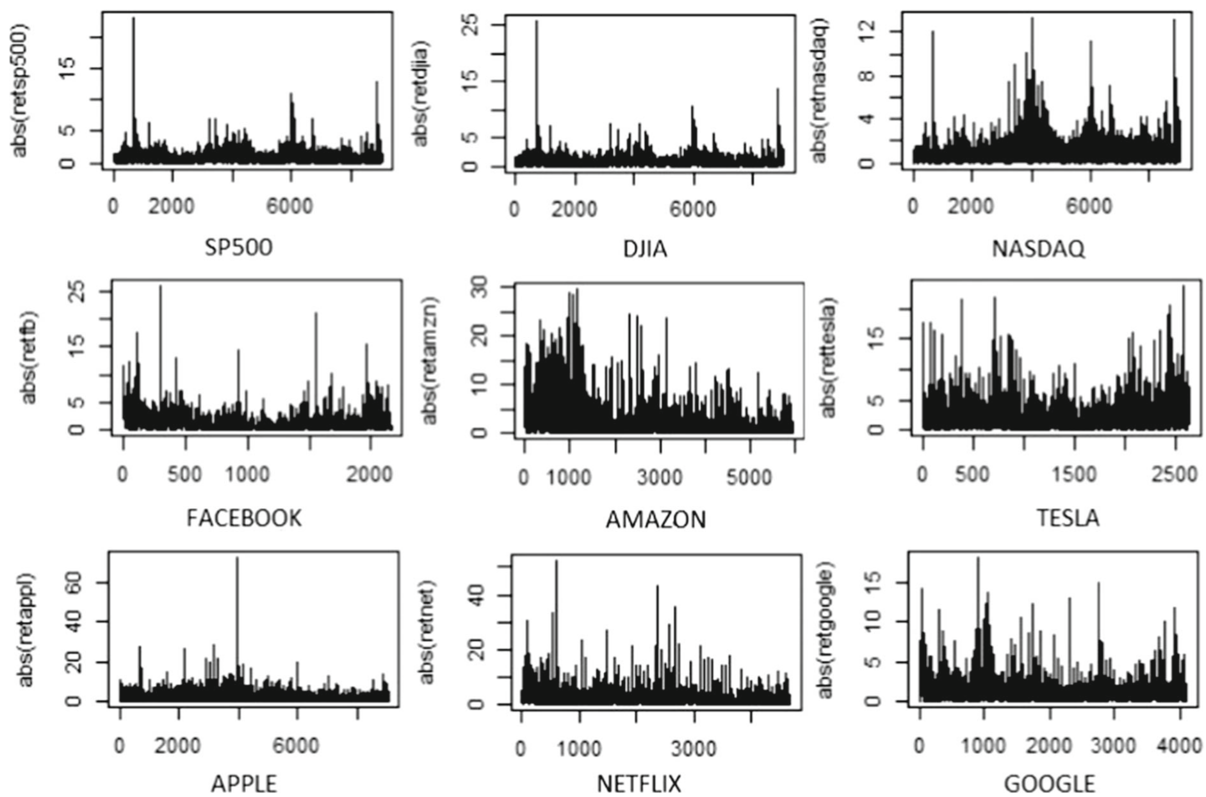

Fig. 2 Daily absolute returns

Table 2 summarizes the basic statistical properties of the data. All the results, with the exception of skewness (which is positive for some series), comply with the stylized facts of returns.

The means of return are all positive but close to zero. (The higher means correspond to the FATANG stocks.) The distribution of returns appears to be somewhat asymmetric as reflected by the negative and positive skewness estimates. All the series returns have heavy tails and strongly depart from normality (skewness and kurtosis coefficients[5] are all statistically different from those of the Normal distribution which are 0 and 3, respectively). The Jarque–Bera normality test statistic is far beyond the critical value (the $p-$value is almost zero) which suggests that r_t is far from a normal distribution for all series.

Figures 2 and 3 plot the two most popular volatility measures regarding the financial asset return: the absolute value and the square of returns ($|r_t|$ and r_t^2) [19]. We can observe the long run behavior of daily $|r_t|$ and r^2, where we can clearly confirm the observation of [29] and [15] that large shocks in financial asset returns tend to be followed by large shocks (of either sign) and small shocks tend to be followed by small shocks.

While market volatility constantly changes over time, it is much higher at times of crisis such as, for example, during the Black Monday stock market crash of 1987, the Russian rouble financial crisis in 1998, the dot-com bubble in which the Nasdaq Composite index peaked in value on March 10, 2000, before crashing, the United States subprime mortgage crisis that occurred between 2007 and 2010, and the COVID-19 pandemic in 2020.

In recent months, and indeed since March 2020 when the COVID-19 turmoil began, we have seen a long period of quiet volatility, especially in the US stock markets. Is this just a cooling-off period for volatility or are the financial asset returns going back to homoskedasticity? To be able to answer this question is one of the main purposes of this investigation.

[5] From now on we just refer skewness and kurtosis.

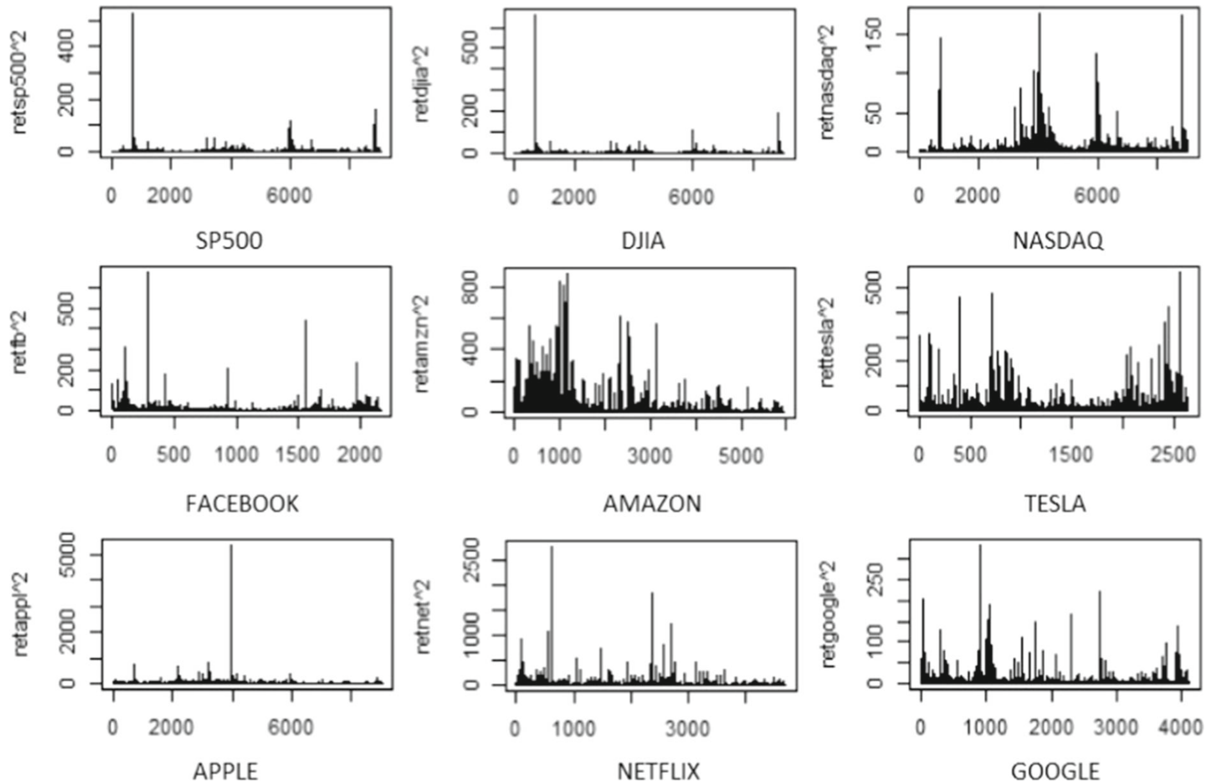

Fig. 3 Daily squared returns

4 Characteristics of the historical returns

The measures presented in Table 2 refer to the whole period under analysis. However, our expectation is that some characteristics of the empirical distributions will have evolved (and changed) over time, especially after the United States subprime mortgage crisis and the COVID-19 pandemic. To notice the temporal changes, all the measures were computed over a rolling window encompassing the previous year of daily observations ($T = 250$). Our first investigation topic relates to the volatility, skewness and kurtosis of the empirical distribution of returns.

4.1 Mean and standard deviation

Have the stock markets been more profitable and less volatile in recent years? And what is the position with regard to the tech stocks? Are they even more profitable and less volatile than the overall market? To answer these questions, we computed the arithmetic average and the standard deviation. In light of the rolling window process, the first value was computed based on the first 250 observations over approximately one year, and the last value, corresponding to December 18, 2020, is the average and variance of returns in 2020. See Figures 4 and 5.

As can be seen, the daily average return is mostly positive (reflecting the trend of increasing prices) but close to zero for all the series under analysis, and higher for FATANG stocks. Due to the higher weighting of these stocks in the composition of the index, the average return of NASDAQ is also higher in comparison with S&P 500 and DJIA. From 2012–2019 to 2020, there was an increase in the average return (except in the DJIA)—with the ratio between the average returns being higher than 1, pointing to a temporal increase in 2020. However, the differences in the averages and distributions (before and after 2020) are not statistically significant—the exception is Tesla, where the daily average return was close to a stunning 0.864% in 2020 (see Table 3 for ANOVA and Kruskal–Wallis tests result). Thus, in spite of the COVID-19 pandemic,

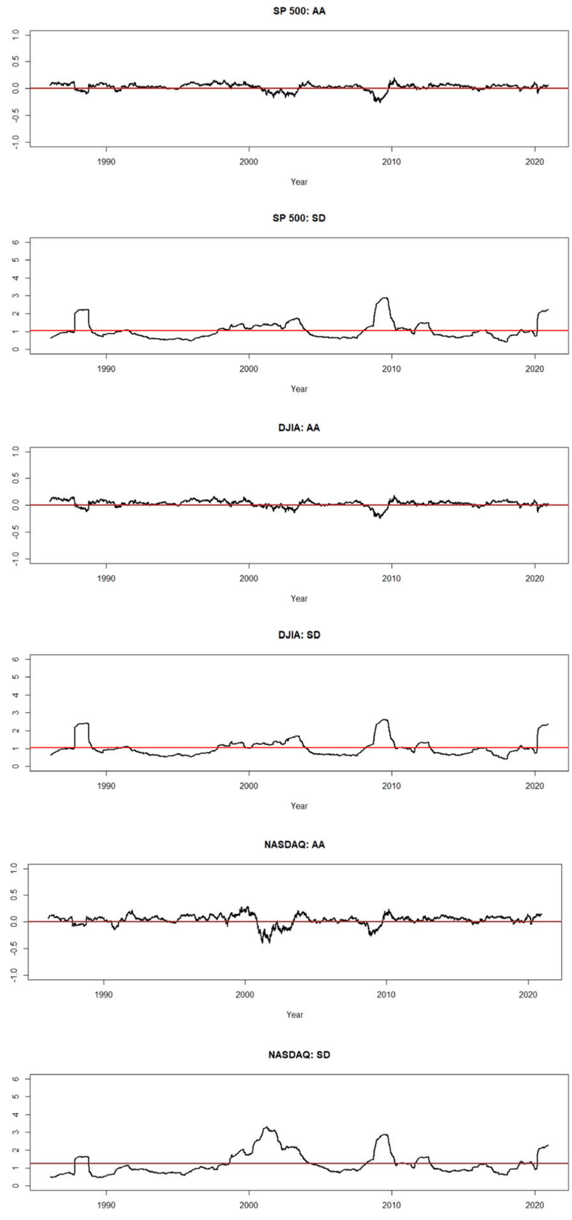

Fig. 4 Rolling window mean (AA) and standard deviation (SD)—US Indices. The arithmetic average (AA) and the standard deviation (SD) are computed over a rolling window encompassing the previous year of daily observations ($T = 250$). The horizontal lines represent the mean of each series

there was an increase (although not statistically significant, with the exception being Tesla) in the daily average return for 8 of the 9 series under scrutiny. The daily average return was 6.48, 2.58 and 2.34 times higher for Tesla, Apple and NASDAQ, respectively. The exception is the DJIA, which has no tech stocks included. The other two stock indices (NASDAQ and S&P 500) also reflect the good performance of FATANG and other tech stocks during the pandemic period.

As expected, the peaks of volatility (measured by the standard deviation) occurred during the most recent crises in the financial markets: the 1987 Black Monday crash, the 1998 Russian rouble financial crisis, the 2000 dot-com bubble, the 2008 subprime mortgage crisis and the 2020 COVID-19 pandemic (see Figs. 4 and 5). We can also observe a sharp decrease in volatility during the years 2012–2019 regardless of the index or the stock being considered. The results of Table 4 confirm that the variance of returns decreased sharply from 2012 to 2019, and increased significantly in 2020[6] (the p value associated with the Levene test[7] is in parenthesis, indicating the rejection of the equality of variances). In 2020, the increase in the variance of returns was higher for the three stock indices (in the case of DJIA, for example, the variance was 8.848 times higher than its variance during the period 2012–2019) when compared to the individual stocks. In the case of Netflix, there was even a decrease in the variance of returns when compared to the variance of the period before. These results show that the increase in volatility was more pronounced in the three stock indices when compared to the individual FATANG stocks. Furthermore, as the increase in the variance was matched by a corresponding increase in the mean, we can conclude that positive returns more than compensate for the negative ones. Thus, the increase in volatility does not necessarily imply a higher risk, as we explain next.

Given these strong differences in volatility, in Sect. 5 we tested for the existence of structural breaks in the unconditional variance of the returns series and the results point to significant breaks.

Next, we also analyzed the relationship between the mean and the variance of returns (per rolling window) in the three periods considered before by estimating the simple linear regression model: $\mu_t = \alpha + \beta\sigma_{rt}^2 + \varepsilon_t$, where the dependent variable is the mean and the independent variable is the variance. Table 5 shows the estimates for β.

As can be seen, the estimates were mostly negative until 2019 (the exception being Tesla). In 2020,

[6] As Facebook was only listed on the New York Stock Exchange after 2012, the cut-off year was 2015.

[7] With regard to departures from normality, the Levene test is less sensitive than the Bartlett test.

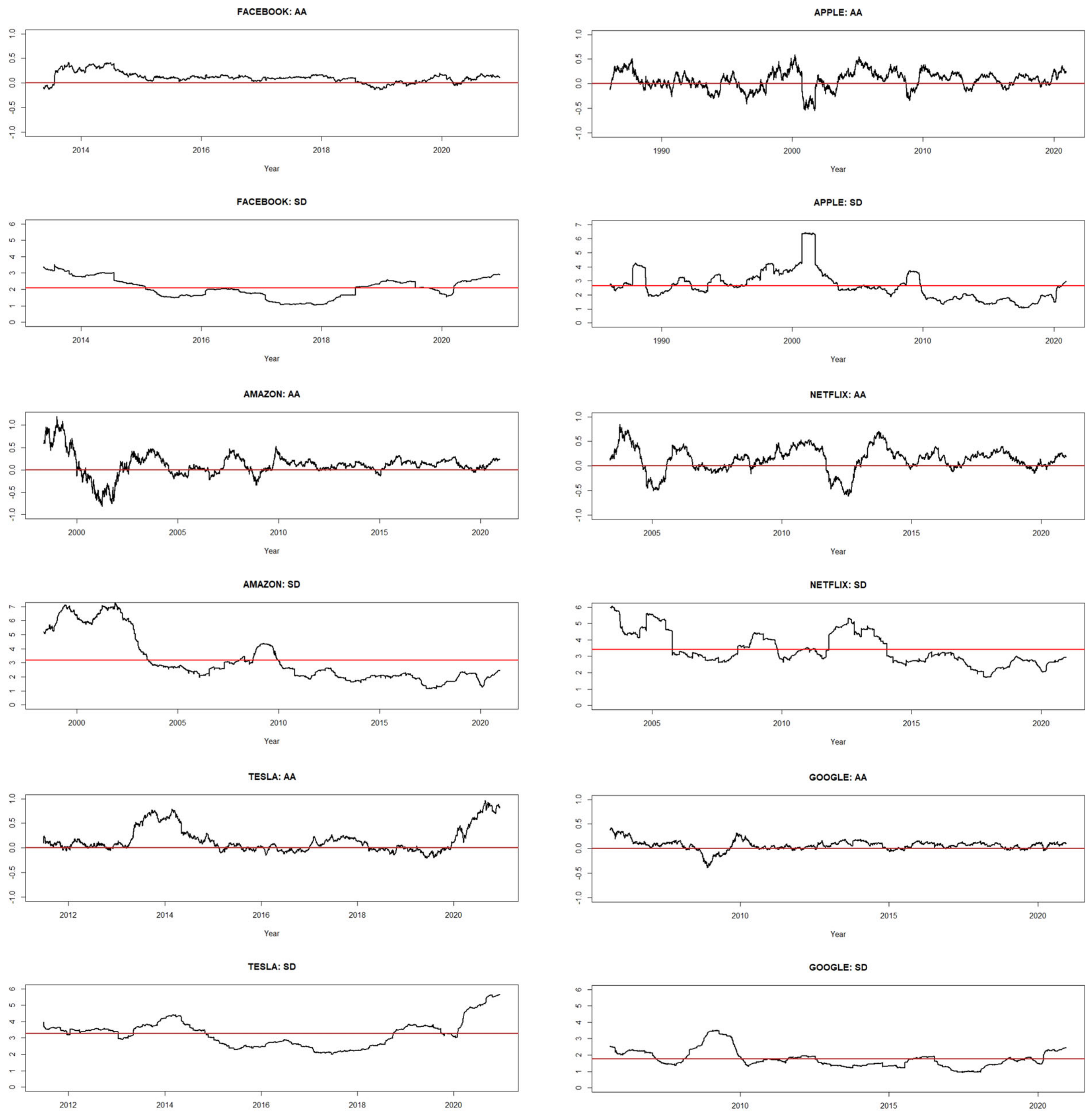

Fig. 5 Rolling window mean (AA) and standard deviation (SD)—FATANG stocks The arithmetic average (AA) and the standard deviation (SD) are computed over a rolling window encompassing the previous year of daily observations ($T = 250$). The horizontal lines represent the mean of each series

most of the estimates were positive (5 out of 8), the estimates are not statistically significant[8] in two cases (Apple and Google), and two estimates were negative (those of the S&P 500 and DJIA stock indices). Thus, for most FATANG stocks (4 out of 6) the average of returns increases with the variance; more volatility seems to indicate greater profitability for most of the financial assets under analysis, highlighting the balance between positive and negative returns, with the first being supreme. Welcome volatility in US stock markets!

[8] Due to potential autocorrelation, Newey–West HAC standard errors are used.

Table 3 Averages comparison before 2012, between 2012 and 2019 and after 2020

	Dec/2011	2012–2019	Ratio	2020	Ratio	ANOVA	K–W
S&P 500	0.029	0.047	1.634	0.056	1.202	0.469 (0.494)	3.896 (0.143)
DJIA	0.033	0.042	1.275	0.023	0.541	0.026 (0.872)	1.033 (0.597)
NASDAQ	0.033	0.061	1.860	0.144	2.336	1.726 (0.189)	9.361 (0.009)
FACEBOOK	0.113	0.075	0.664	0.121	1.625	0.009 (0.923)	0.361 (0.835)
AMAZON	0.122	0.118	0.967	0.224	1.906	0.047 (0.828)	0.687 (0.158)
TESLA	0.048	0.133	2.800	0.864	6.477	6.229 (0.013)	8.314 (0.016)
APPLE	0.070	0.088	1.246	0.226	2.580	0.448 (0.503)	2.675 (0.263)
NETFLIX	0.087	0.173	1.983	0.205	1.182	0.686 (0.408)	0.765 (0.682)
GOOGLE	0.100	0.071	0.707	0.105	1.486	0.085 (0.770)	2.262 (0.323)

K–W: Kruskal–Wallis test (p value is in parenthesis). Ratio is the division of average return between two periods. For example, the ratio after column "2020" divides the average return for the year 2020 by the daily average return of the period 2012–2019. The columns "Dec/2011", "2012–2019" and "2020" include the daily average return in each period

Table 4 Variances comparison before 2012, between 2012 and 2019 and after 2020

	Until Dec/2011	2012–2019	Ratio	2020	Ratio	Levene test
S&P 500	1.431	0.655	0.457	4.925	7.525	109.08 (0.000)
DJIA	1.365	0.635	0.465	5.616	8.848	128.39 (0.000)
NASDAQ	2.135	0.938	0.439	5.248	5.598	84.59 (0.000)
FACEBOOK	8.737	3.193	0.365	8.606	2.695	61.66 (0.000)
AMAZON	19.202	3.459	0.180	5.985	1.730	243.99 (0.000)
TESLA	13.910	9.749	0.701	32.523	3.336	63.98 (0.000)
APPLE	9.611	2.604	0.271	8.830	3.391	178.23 (0.000)
NETFLIX	17.029	9.239	0.543	8.649	0.936	37.78 (0.000)
GOOGLE	5.033	2.123	0.422	6.021	2.836	79.67 (0.000)

Ratio is the division of the variance from two different periods. For example, the ratio after column "2020" divides the returns' variance of the year 2020 by the variance of the period 2012–2019. The columns "Until Dec/2011", "2012–2019" and "2020" include the variance for each period. Levene test p value is in parenthesis

Table 5 Mean-variance regression

Index/stock	Until Dec/2011	2012–2019	2020
S&P 500	– 0.0285*	– 0.0365*	– 0.0058*
DJIA	– 0.0298*	– 0.0473*	– 0.0071*
NASDAQ	– 0.025*	– 0.0449*	0.0076*
FACEBOOK	– 0.0028	– 0.0295*	0.0088*
AMAZON	– 0.0014*	– 0.0248*	0.0516*
TESLA	0.0414*	0.0266*	0.0301*
APPLE	– 0.0119*	– 0.0685*	0.0016
NETFLIX	– 0.0081*	– 0.0131*	0.0536*
GOOGLE	– 0.0258*	– 0.0137*	– 0.0004

Estimates for β in the simple linear regression model: $\mu_t = \alpha + \beta\sigma_{rt}^2 + \varepsilon_t$, where μ_t and σ_{rt}^2 represent the rolling window mean and variance. *Denote statistically significant at the 1% significance level, based on the p value associated with the corresponding t significance test

4.2 Skewness and kurtosis

Another common stylized fact of financial assets is that large negative returns occur more often than equally large upward movements, which leads to the negative asymmetry of the empirical distributions [7]. In the recent past, however, large positive returns have also occurred. This has reversed the type of asymmetry, especially regarding the FATANG stocks. In order to test this empirical fact, a rolling window encompassing the previous year's daily returns was used to estimate the coefficients of skewness and kurtosis. The results are shown graphically in Figs. 6 and 7.

As can be seen, when the stock indices are considered, most of the estimates of skewness are negative, with only a few being statistically significant and positive. Furthermore, the peaks in the kurtosis are mostly due to large negative returns, giving rise to the negative spikes of the coefficient of skewness. Thus, the number of large negative returns exceeds that of positive returns, leading to the negative skewness of the empirical distributions. The conclusions are different when the individual FATANG stocks are analyzed. The estimates of skewness can be either positive or negative, and the peaks of kurtosis are also explained by large positive returns, leading to the positive spikes of the coefficient of skewness. Thus, large positive returns also inflate the kurtosis in the case of the FATANG stocks.

The ratio (percentage) between the semi-kurtosis and the kurtosis (see Sect. 2.1), computed in accordance with the 250-day rolling window method, is represented graphically in Figs. 8 and 9). The horizontal line represents 50%.

If we take the variance/standard deviation as a measure of risk, we conclude, based on Table 4, that FATANG stocks are riskier when compared to the US stock indices because the value is higher. However, as one can see in Table 6, the value of the ratio is always higher than 70% for the three US stock indices. Thus, the downside "dominates" the upside in more than 70% of the rolling windows. In the case of stocks, it is the opposite, thus highlighting the importance of large positive returns as opposed to negative ones. Whereas the volatility and kurtosis of Facebook, Amazon, Netflix and Google stock returns represent more uncertainty than risk, they represent more risk than uncertainty in the case of stock indices. Thus, the indices seem to be riskier when compared to this particular class of tech stocks. (In the case of Tesla and Apple, the ratio between semi-kurtosis and kurtosis is still higher than 50% in more that 50% of the rolling windows, but less than 20 percentage points when compared to the US stock indices.) The chance to sell at a lower price than their purchase price seems to be higher in the case of stock indices. Thus, if the distributions are asymmetrical, the ratio between semi-kurtosis and kurtosis seems more suitable than the variance/standard deviation as a measure of risk.

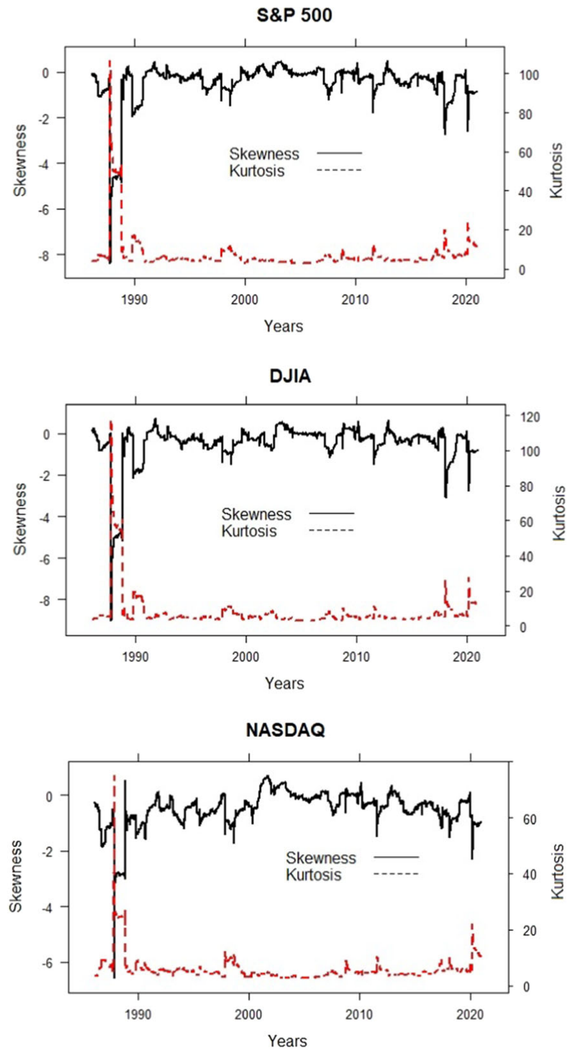

Fig. 6 Skewness and kurtosis—US indices

4.3 Autocorrelation analysis of the return series

It is well established that the serial correlation (or autocorrelation) of financial asset returns is often insignifi-

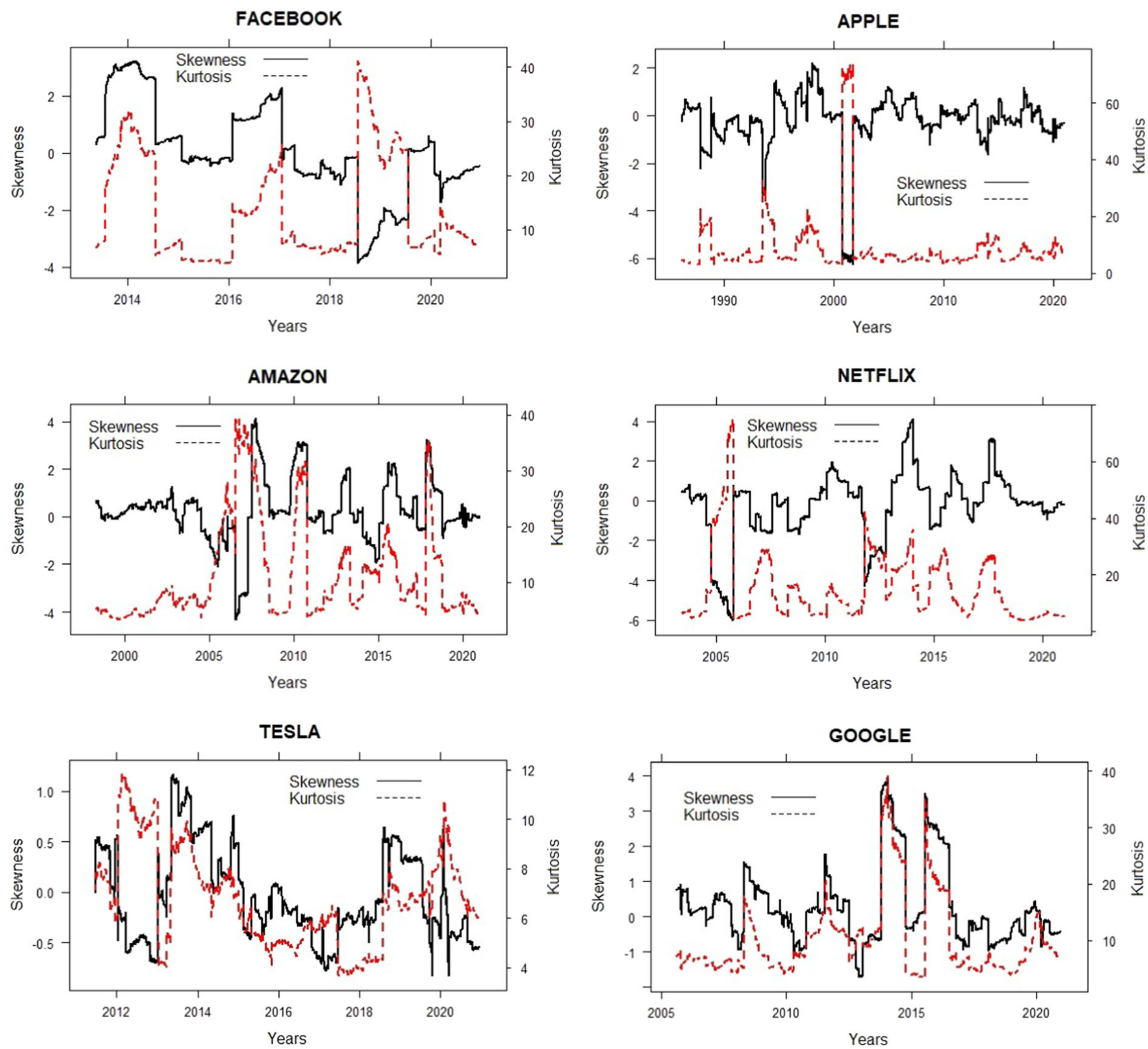

Fig. 7 Skewness and kurtosis—FATANG stocks

cant [16,19,52] giving empirical support to the weak-form of the efficient market hypothesis (EMH), which states that past returns cannot be used to predict future returns. Thus, the returns are practically white noise [19]. To conclude with regard to the autocorrelation of returns, absolute returns and square returns over time, the Ljung–Box (LB) test for up to the tenth-order serial correlation is computed over a rolling window encompassing the previous year's daily observations ($T = 250$). The evolution of the LB test value is represented graphically in Figures 10, 11 and 12. The horizontal line represents the critical value for a 1% significance level: $\chi^2_{10} = 23.20925$.

As can be seen, and according to the Ljung–Box statistic for returns, the autocorrelation seems not to be relevant (in only a small percentage of windows is the "no autocorrelation" null hypothesis rejected, the exception being the NASDAQ composite index with more than 20% of rejections; see Table 7 where the number of rejections and the respective percentages are shown). The autocorrelation structure of returns still seems weaker in the case of individual stocks (in the case of Tesla, for example, the number of rejections is

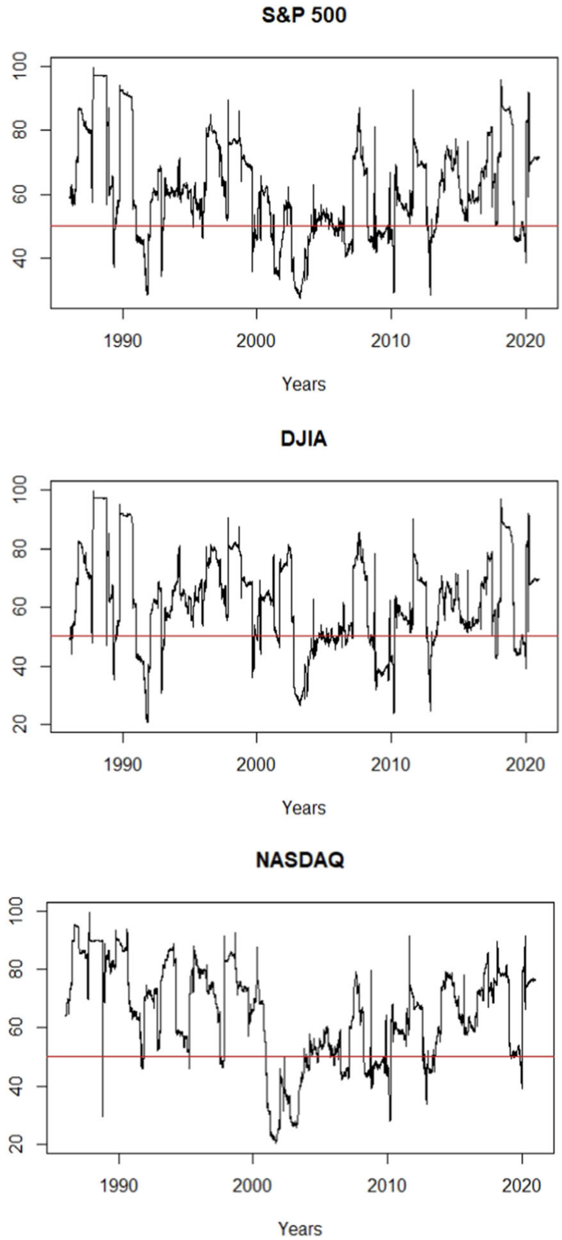

Fig. 8 Ratio of the semi-kurtosis in the kurtosis—US indices

very close to zero). In general, the rejection of the null hypothesis is due to the statistical significance of the positive first-order and negative second-order autocorrelations. (This supports the so-called mean-reversion behavior of stock market returns.) In spite of its significance, the absolute value of the first- and second-order autocorrelations is very small, suggesting that returns (r_t) do have some memory, albeit a very short one, and there is a portion of stock market returns that is predictable although it might be a very small one. Therefore, the efficient market or random walk hypothesis does not strictly hold [12].

Even though the series of returns seem to be weakly correlated over time, the autocorrelation of absolute and squared returns is stronger, pointing to a positive autocorrelation over several days, which quantifies the fact that high volatility events tend to cluster in time [7]. However, the volatility clustering stylized fact is not so evident for individual stocks, especially in the cases of Facebook, Amazon and Netflix. The empirical results seem also to confirm that sample autocorrelations for absolute returns are greater than the sample autocorrelations for squared returns.

Conditional heteroskedasticity of returns is a possible explanation for the large positive autocorrelation between $|r_t|$ and r_t^2, i.e., the variance or conditional variance is not constant, and it changes over time. The ARCH type model is the most important class of nonlinear time series model that is able to capture some aspects of the time varying volatility structure. The Lagrange multiplier test proposed by [13] can be used to formally test the presence of conditional heteroskedasticity and the evidence of ARCH effects. The LM test for a twelve-order linear ARCH effect is computed over a rolling window encompassing the previous year's daily observations ($T = 250$). An ARMA(4,0) model is first of all estimated to pre-filter the data from linear dependency. Figures 13 and 14 show the temporal evolution of the ARCH LM test for indices and stocks. The horizontal line represents the critical value for a 1% significance level: $\chi^2_{12} = 26.22$.

The ARCH-LM test results suggest, as is common in empirical finance, that all the returns series under analysis exhibit ARCH effects, inferring that nonlinearities should enter through the variance of the processes [22]. ARCH or GARCH models can be used to capture such behavior, by conditioning the volatility of the process on past information. In Sect. 5, we use the ARMA-GARCH models to describe the conditional distribution of returns. However, it seems that conditional heteroskedasticity is more evident for stock indices, and the ARCH effect has declined in the most recent past (the exception being the year 2020), especially in the case of FATANG stocks. Thus, in the most recent past, market volatility did not change so sharply over time, which brings us back to the question in the title "To keep faith with homoskedasticity or to go back

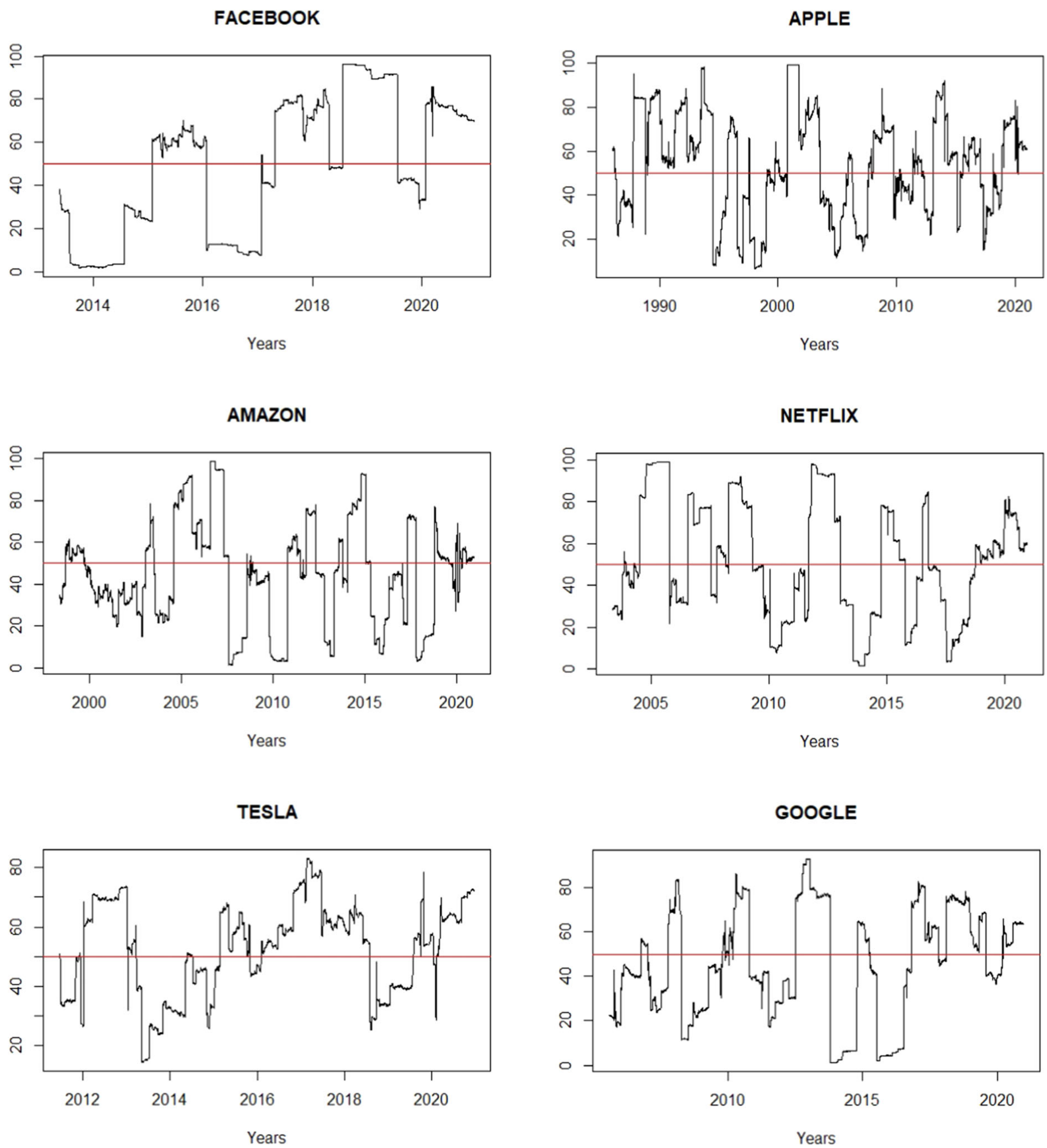

Fig. 9 Ratio of the semi-kurtosis in the kurtosis—FATANG stocks

to heteroskedasticity?". In order to check this, we first computed the number of 250-day windows where the null hypothesis of the ARCH-LM test was rejected. The results are shown in Table 8.

As can be seen, the percentage of rejections is less than 20% in the case of FATANG stocks, and around 40% in the case of US stock indices. Thus, the returns of US stock indices are more conditionally heteroskedastic, while the FATANG tech stocks returns are characterized mostly by conditional homoskedasticity. In both cases, the percentage of rejections is lower than 50%: the number of non-rejections of conditional

Table 6 Number of ratios above 50%

Indices/stock	# Ratio > 50%	# Obs	%
S&P 500	6608	9047	73.04
DJIA	6630	9047	73.28
NASDAQ	6990	9047	77.26
FACEBOOK	986	2161	45.63
AMAZON	2350	5939	39.57
TESLA	1477	2637	56.01
APPLE	5132	9047	56.73
NETFLIX	2138	4677	45.71
GOOGLE	1671	4113	40.63

homoskedasticity assumption exceeds the number of rejections. However, this raises a new million dollar question: after the COVID- 19 turmoil, how long will it remain like this?

5 Volatility clustering, persistence and asymmetry

As we mentioned previously, our main purpose is to reach conclusions concerning the characteristics of volatility over a long period (focusing mainly on the last years), by comparing FATANG stocks with three US stock indices. As the international financial markets are intermittently subject to shocks resulting from global economics, wars, political events and investors' decisions, these shocks can cause sudden breaks in the unconditional variance of returns. These sudden breaks are equivalent to structural breaks in the parameters of the returns' conditional volatility processes.

To identify and determine the structural breaks in the volatility of the nine returns series, we computed a modified version of the [23] iterated cumulative sum of squares (ICSS) algorithm that allows for the conditional dependent processes proposed by Rapach and Strauss [40]. The PELT algorithm [24] was also used, and the results are similar. The number of each observation where each break occurs and the exact dates of the structural breaks are reported in Table 9.

The variances ("Var" in the table) are computed for each of the sub-samples defined by (and up to) the structural breaks identified by the modified ICSS algorithm. As one can see, there is strong evidence of structural breaks in the unconditional variance for all the returns series, leading to distinct regimes in volatility. The ICSS algorithm selects six structural breaks in the unconditional variance for the S&P 500 and DJIA returns; five structural breaks for NASDAQ and Amazon; four structural breaks for Apple and Google and three structural breaks for Facebook, Netflix and Tesla.

As expected, the contemporaneity of structural breaks occurs during the most recent crises in the financial markets: 1998 Russian rubble financial crisis, 2000 dot-com bubble, 2008 subprime mortgage crisis and 2020 COVID-19. The oldest crises did not reach some of the FATANG stocks because they were not listed yet at that time (see Table 2). The behavior of S&P, DJIA, NASDAQ, APPLE and GOOGLE is very similar: after subprime crisis the algorithm detected only one structural break due to COVID-19. Thus, they remained in the same volatility regime for more than 10 years. The last structural break for Netflix, Amazon and Facebook was detected in 2013, 2017 and 2018, respectively. Netflix reached the lowest levels of volatility after 2013.

The unconditional variance in the sub-sample resulting from the penultimate structural break (in bold) is smaller in comparison with the variance of most of the other sub-samples. This also confirms the smaller volatility in the years before February 2020, as we pointed out previously. The variance increases sharply in the last sub-sample (the exception being Netflix, where there is even a decrease in variance), influenced particularly by the higher volatility of 2020.

COVID-19 gave rise (in February 2020) to a new volatility regime in S&P 500, DJIA, NASDAQ, Apple, Google and Tesla. (The structural break is more or less contemporaneous for the indices and these stocks.) However, the increase in volatility was not enough to change the regime in the returns volatility of Amazon, Facebook and Netflix. (For these stocks, the last

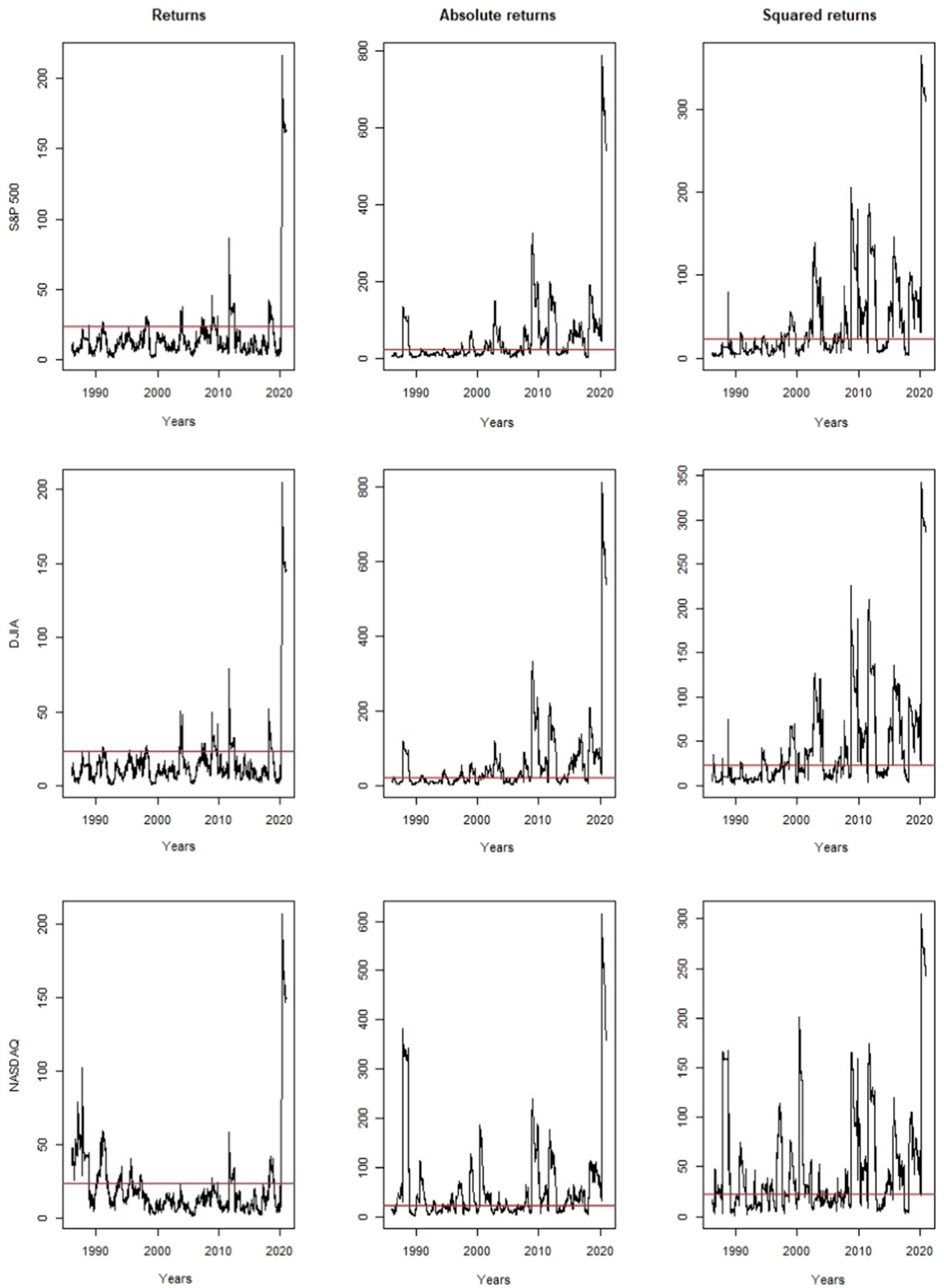

Fig. 10 Ljung–Box statistic—US indices

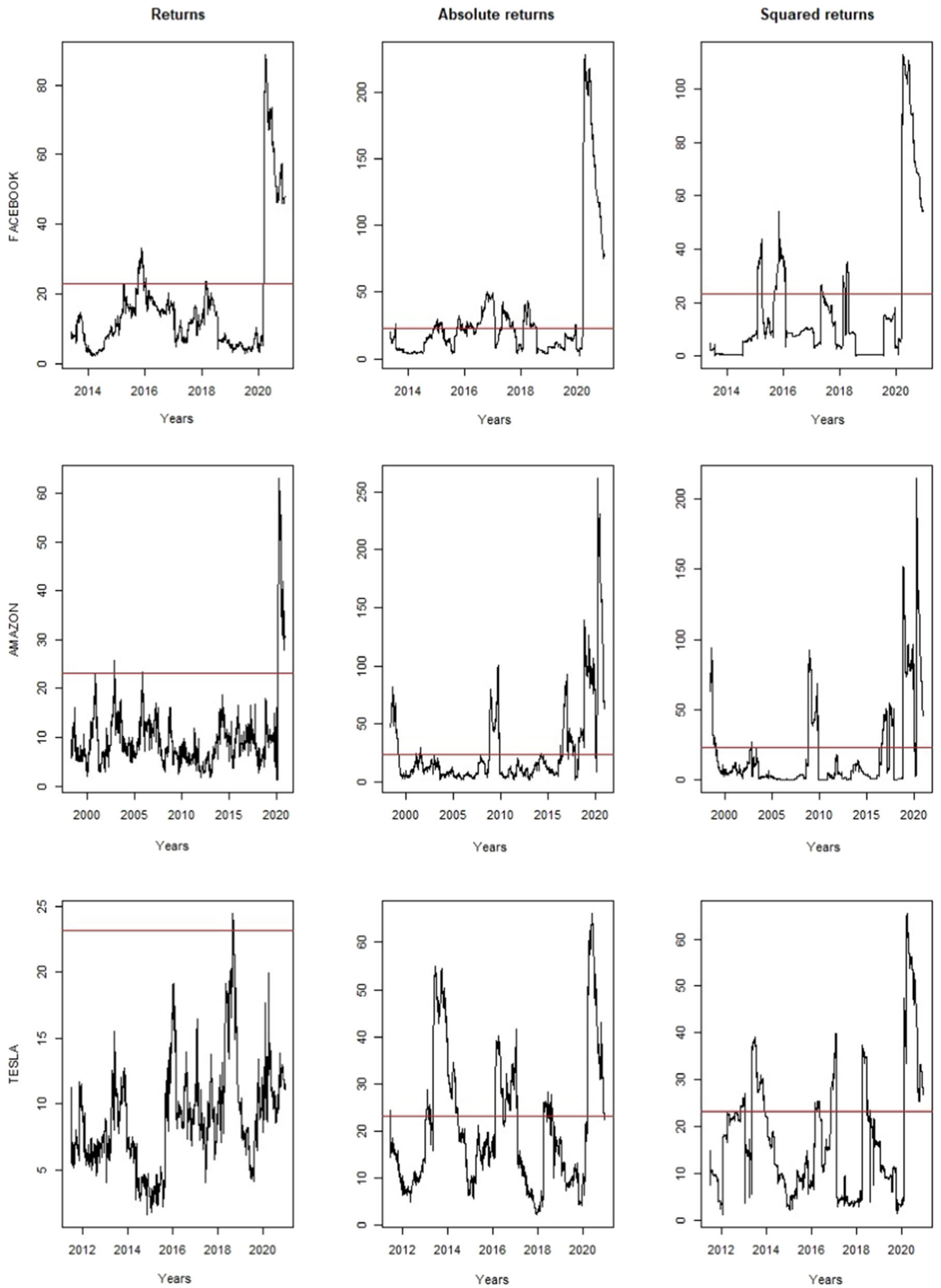

Fig. 11 Ljung–Box statistic—FATANG stocks

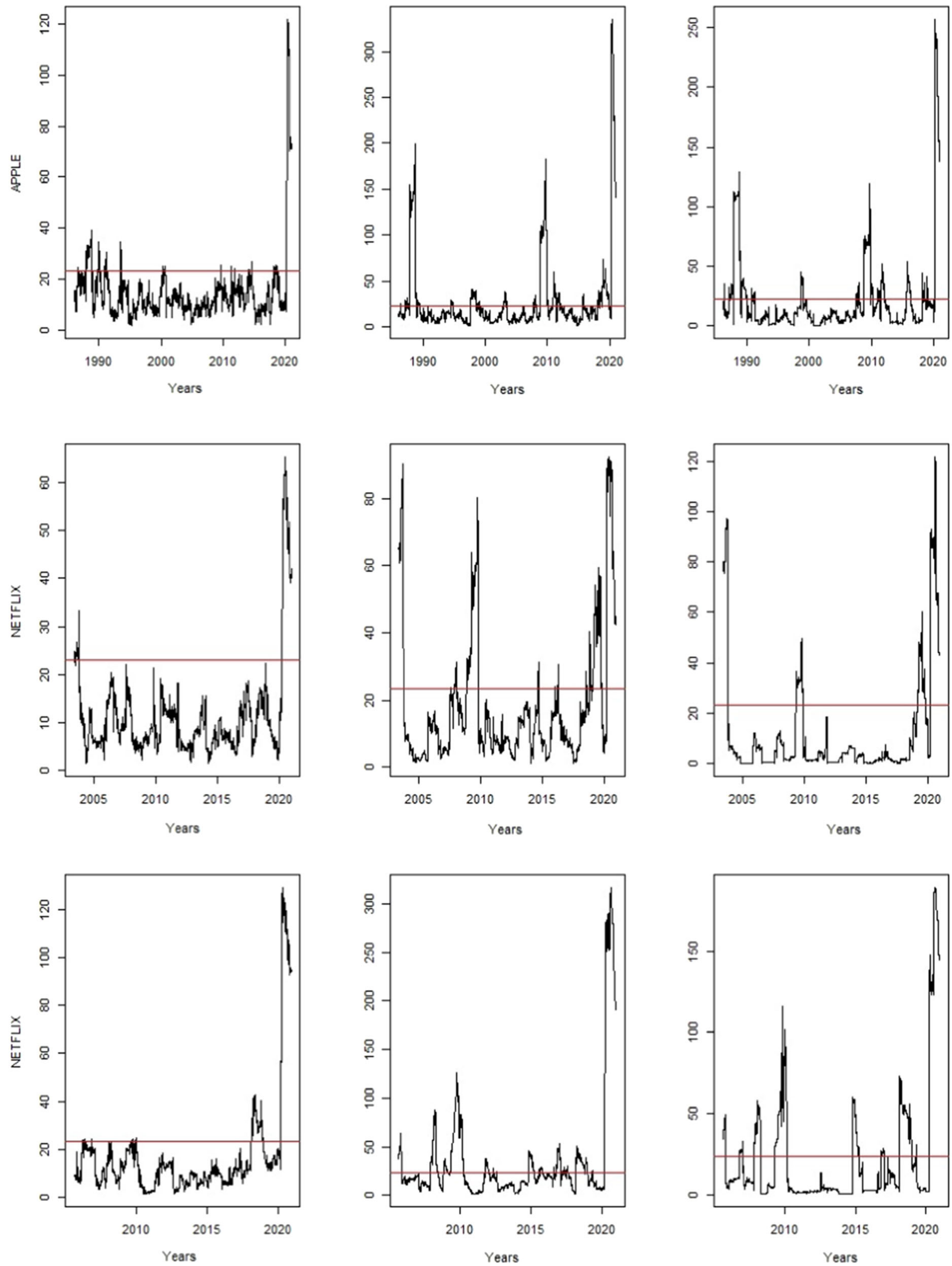

Fig. 12 Ljung–Box statistic—FATANG stocks

Table 7 Number of rejections of the null hypothesis in the Ljung–Box test

Indices/stocks	r_t		$\|rt\|$		r_t^2	
	# Rejections	%	# Rejections	%	# Rejections	%
S&P 500	1158	13.16	4070	46.27	4105	46.66
DJIA	1188	13.50	4726	53.72	4220	47.97
NASDAQ	2138	24.30	5127	58.28	5209	59.21
FACEBOOK	259	13.55	736	38.51	396	20.72
AMAZON	199	3.50	1500	26.37	1194	20.99
TESLA	13	0.54	832	34.86	590	24.72
APPLE	657	7.47	2241	25.47	2052	23.33
NETFLIX	280	6.32	872	19.70	598	13.51
GOOGLE	426	11.03	1309	33.89	1043	27.00

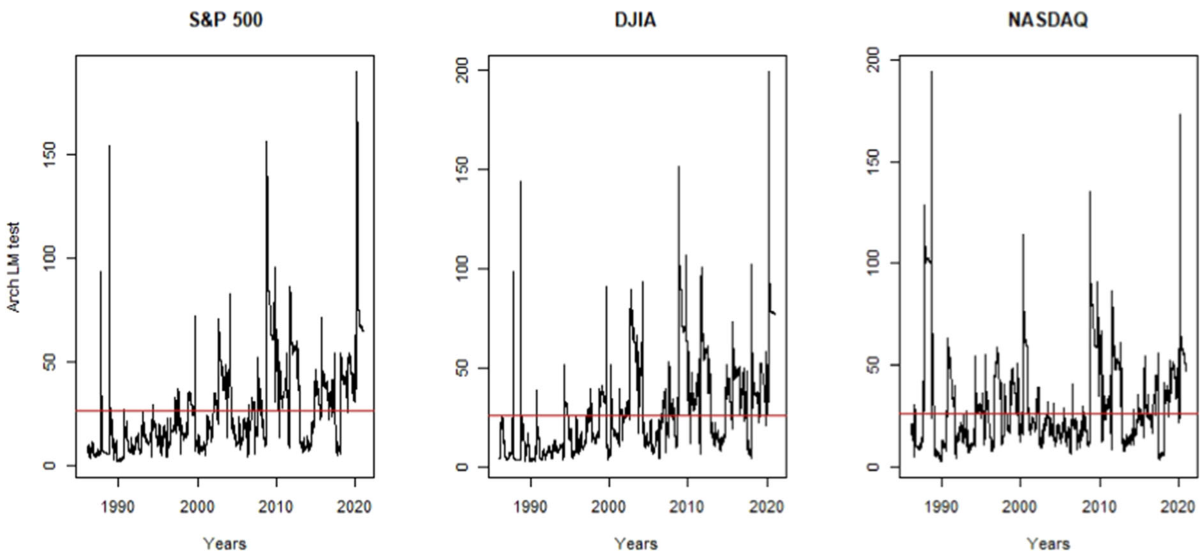

Fig. 13 ARCH LM statistic—US Indices

break is very far from the COVID-19 shock.) It would appear, therefore, that 50% of the FATANG stocks were immune to the virus. Thus, we can bring up question again: "To keep faith with homoskedasticity or to go back to heteroskedasticity?" Even for the six financial assets with structural breaks in volatility, was COVID just an accident and they will quickly return to the previous regime of almost constant variance?

Next, we analyzed the conditional distribution of returns to get further clarification on volatility. We estimated an ARMA(4,0) model for the conditional mean in combination with the GARCH(1,1), GJR(1,1) and EGARCH(1,1) models for the conditional variance across the sub-samples that resulted from the structural breaks identified by the modified ICSS algorithm. To better account for leptokurtosis, we consider the Student's t distribution for innovations instead of the standardized normal. The estimation results across the sub-samples are shown in Tables 10 through 18. The analysis of results will be divided according to the three main stylized facts of the volatility of returns: volatility clustering, persistence, and asymmetric effect.

We focused firstly on the ARCH-LM test results. As can be seen, we rejected the null hypothesis of the ARCH-LM test, indicating the existence of conditional heteroskedasticity in most of the series and for most of the sub-samples resulting from the structural breaks. This conclusion confirms the stylized fact of

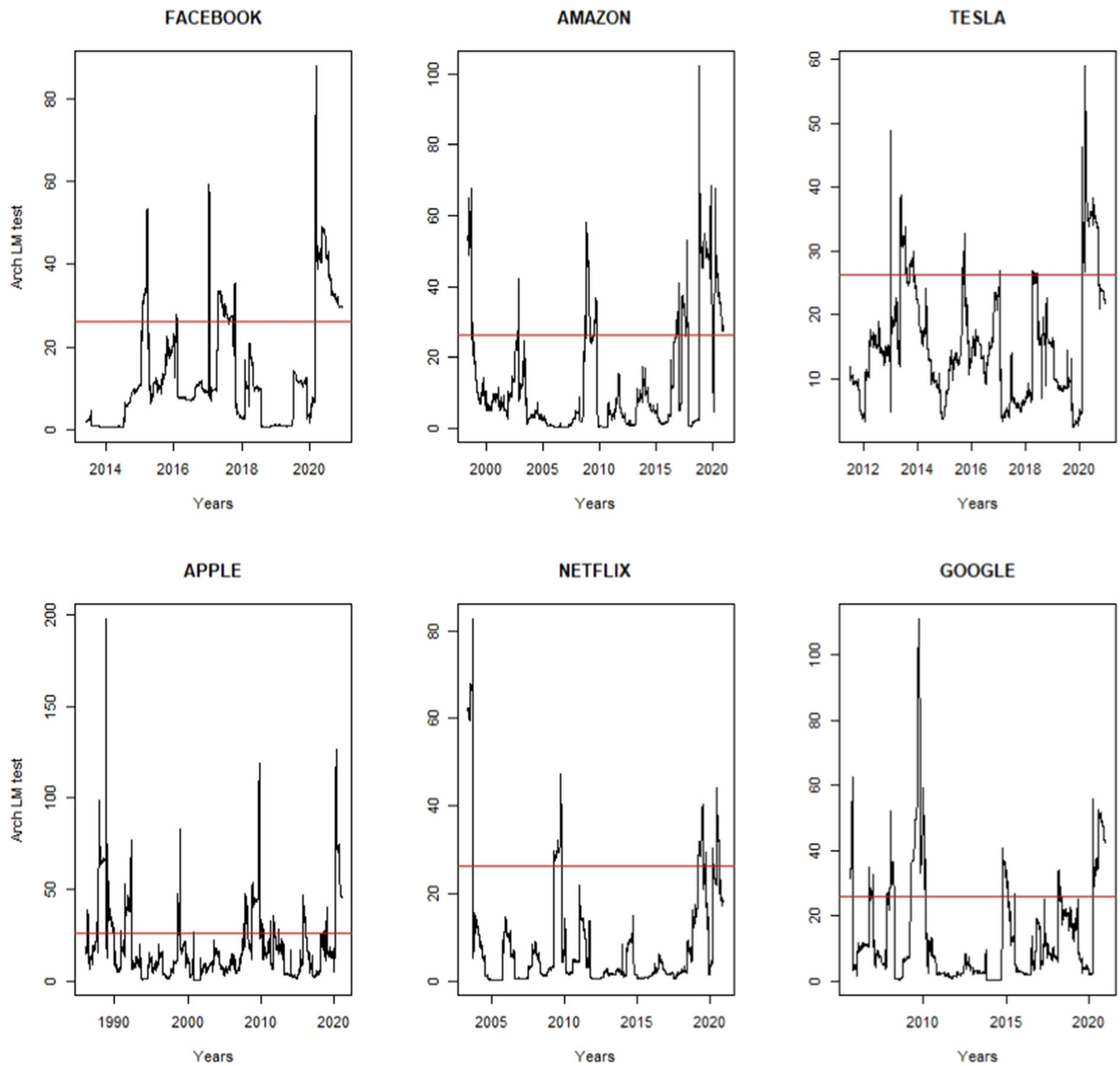

Fig. 14 ARCH LM statistic—FATANG stocks

the returns volatility clustering. However, among the FATANG stocks the decision was not the same for all the sub-samples. The stock returns of Facebook, Netflix (in all the sub-samples) Amazon, Apple, Tesla and Google (in some of the sub-samples) were characterized by conditional homoskedasticity. Furthermore, in the case of Google and Tesla, and for recent years, conditional homoskedasticity was rejected only in the sub-sample that includes the COVID-19 turmoil. Before that, and for long periods of time, the null of the ARCH-LM test was not rejected. Thus, the suspicion of constant variance of returns is more evident among the FATANG stocks, which calls into question the existence of volatility clustering.

And what about volatility persistence? According to [14], in a GARCH(1, 1) model, there are two main consequences if $\alpha + \beta = 1$: persistence on the conditional variance forecasting; and the variance of the unconditional distribution of the error term ε_t is infinite. In other words, when $\alpha + \beta = 1$ current shock persists indefinitely by conditioning the future variance. Thus, and according to [39], a significant impact of volatility on

Table 8 ARCH-LM test: number of rejections

Indices/stocks	# Rejections	# Obs	%
&P 500	3360	8797	38.19
DJIA	3592	8797	40.83
NASDAQ	3864	8797	43.92
FACEBOOK	354	1911	18.52
AMAZON	982	5689	17.26
TESLA	303	2387	12.69
APPLE	1729	8797	19.65
NETFLIX	375	4427	8.47
GOOGLE	713	3863	18.46

Table 9 Structural breaks in the unconditional variance

Indices/stock	Obs number	Date	Var	Indices/stock	Obs number	Date	Var
S&P 500	754	25/01/1988	14.17	DJIA	744	11/01/1988	17.66
	3073	26/03/1997	0.58		3063	12/03/1997	0.42
	4665	25/07/2003	1.81		4665	25/07/2003	1.70
	5669	23/07/2007	0.46		5669	23/07/2007	0.43
	6783	20/12/2011	3.12		6137	01/06/2009	4.32
	8837	21/02/2020	0.65		8837	21/02/2020	0.79
			5.64				6.42
NASDAQ	3409	27/07/1998	0.83				
	4586	02/04/2003	5.86				
	5958	12/09/2008	1.20				
	6109	21/04/2009	12.07				
	8836	20/02/2020	1.17				
			5.95				
FACEBOOK	496	12/05/2014	10.63	AMAZON	1313	06/08/2002	38.82
	991	28/04/2016	3.32		2499	24/04/2007	6.80
	1421	11/01/2018	1.26		3132	26/10/2009	14.09
			5.93		4777	10/05/2016	4.36
					5147	26/10/2017	1.26
							5.93
TESLA	970	08/05/2014	14.29	APPLE	3139	30/06/1997	12.46
	1947	26/03/2018	5.71		3962	03/10/2000	997.48
	2411	28/01/2020	12.21		6133	26/05/2009	8.75
			34.21		8836	20/02/2020	2.63
							9.78
NETFLIX	605	18/10/2004	31.27	GOOGLE	436	12/05/2006	6.37
	2345	14/09/2011	10.71		811	07/11/2007	2.11
	2747	23/04/2013	26.85		1115	23/01/2009	10.82
			7.01		3902	20/02/2020	2.30
							6.69

Table 10 GARCH, GJR and EGARCH(1,1), persistence and Half-life—S&P 500

	Full sample	25/01/1988	26/03/1997	25/07/2003	23/07/2007	20/12/2011	21/02/2020	18/12/2020
ARCH-LM	980.282*	15.224	51.079*	107.330*	35.930*	335.662*	250.550*	56.268*
α	0.093*	− 0.081	0.026*	0.076*	− 0.031	0.104*	0.191*	0.172*
β	0.902*	1.080	0.969*	0.888*	0.008	0.895*	0.774*	0.819*
$\alpha + \beta$	0.995	0.999	0.995	0.964	NA	0.999	0.965	0.991
Half-life	130.76	692.801	138.283	18.905	NA	692.801	19.654	76.704
t-df	5.508*	9.795*	5.227*	9.242*	8.530*	6.601*	5.067*	5.810*
γ GJR	0.141**	0.109***	0.018***	0.175*	0.159*	0.172*	0.367*	0.024
γ EGARCH	− 0.108*	− 0.142**	− 0.029*	− 0.141*	− 0.127*	− 0.149*	− 0.253*	− 0.048
BIC GARCH	2.606	5.079	2.159	3.352	2.098	3.526	2.168	3.839
BIC GJR	2.590	5.166	2.160	3.313	2.058	3.495	2.122	3.865
BIC EGARCH	2.585	5.165	2.159	3.308	2.056	3.496	2.110	3.875

ARCH-LM test. *, **, ***Denote statistically significant at the 1%, 5% and 10% significance levels, respectively. $\alpha + \beta = 1$ is the measure of volatility persistence. Half-life gives the point estimate of the half-life in days given as $HL = \frac{\log(0.5)}{\log(\alpha+\beta)}$. `t-df` represents the Student's t degrees of freedom. GARCH, GJR and EGARCH are conditional heteroskedastic models defined in (4), (6) and (5), respectively

Table 11 GARCH, GJR and EGARCH(1,1), persistence and half-life—DJIA

	Full sample	11/01/88	30/12/91	12/03/97	25/07/03	23/07/07	01/06/09	21/02/2020	18/12/2020
ARCH-LM	751.758*	33.710*	6.607	28.331*	119.160*	27.039*	149.504*	419.496*	66.738*
α	0.091*	0.035*	− 0.018	0.046*	0.070*	0.043**	0.090*	0.164*	0.213*
β	0.900*	0.937*	1.017*	0.914*	0.890*	0.910*	0.898*	0.819*	0.785*
$\alpha + \beta$	0.992	0.972	0.999	0.960	0.960	0.953	0.988	0.984	0.997
Half-life	85.630	24.407	692.801	16.98	16.98	14.398	57.415	42.418	272.224
t-df	5.555*	3.919*	6.622*	7.578*	8.44*	9.908*	21.077*	5.056*	8.447*
γ GJR	0.125*	0.022	0.028	0.914*	0.134*	0.168*	0.148*	0.305*	0.056
γ EGARCH	− 0.097*	− 0.025	− 0.056**	− 0.077*	− 0.113**	− 0.151*	− 0.124*	− 0.207*	− 0.055
BIC GARCH	2.589	2.731	2.789	1.968	3.269	2.029	3.987	2.309	3.969
BIC GJR	2.576	2.736	2.804	1.970	3.248	2.003	3.961	2.268	3.993
BIC EGARCH	2.571	2.735	2.803	1.969	3.236	1.997	3.960	2.266	4.000

ARCH-LM test. *, **, ***Denote statistically significant at the 1%, 5% and 10% significance levels, respectively. $\alpha + \beta = 1$ is the measure of volatility persistence. Half-life gives the point estimate of the half-life in days given as $HL = \frac{\log(0.5)}{\log(\alpha+\beta)}$. `t-df` represents the Student's t degrees of freedom. GARCH, GJR and EGARCH are conditional heteroskedastic models defined in (4), (6) and (5), respectively

stock prices can only take place if shocks to volatility persist over a long time.

The parameter estimates resulting from the different models reveal that the GARCH(1,1) processes are highly persistent (almost-integrated) when the full sample is considered, with the $\alpha + \beta$ estimate ranging from 0.989 to 1.000, in line with the extant literature. However, persistence in the full-sample estimates, as pointed out by Rapach and Strauss [40], sometimes masks important differences in persistence across sub-samples. If we consider all the sub-samples, the estimates range between 0.207 (Facebook) and almost 1.000 (NASDAQ).

Table 19 shows the estimates for the two most recent sub-samples. As can be seen, persistence is always higher among the US stock indices, taking longer to

Table 12 GARCH, GJR and EGARCH(1,1), persistence and half-life—NASDAQ

	Full sample	27/07/1998	02/04/2003	12/09/2008	21/04/2009	20/02/2020	18/12/2020
ARCH-LM	2046.298*	1013.827*	116.000*	108.560*	33.232*	391.490*	40.722*
α	0.111*	0.115*	0.096*	0.038*	– 0.090***	0.131*	0.172**
β	0.889*	0.860*	0.866*	0.956*	– 0.605**	0.846*	0.828*
$\alpha + \beta$	0.999	0.975	0.962	0.994	– 0.695	0.977	1.000
Half-life	830.765	27.378	17.892	115.178	NA	29.802	1571.416
t-df	6.726*	5.588*	34.853*	18.935*	89.826*	5.247*	3.891*
γ GJR	0.120*	0.104*	0.178*	0.061*	0.229	0.303*	0.272*
γ EGARCH	−0.087*	– 0.079*	– 0.125*	– 0.050*	– 0.316***	– 0.226*	– 0.122
BIC GARCH	2.909	2.234	4.527	2.975	5.508	2.763	4.157
BIC GJR	2.900	2.231	4.496	2.964	5.557	2.723	4.156
BIC EGARCH	2.897	2.227	4.502	2.965	5.539	2.715	4.175

ARCH-LM test. *, **, ***Denote statistically significant at the 1%, 5% and 10% significance levels, respectively. $\alpha + \beta = 1$ is the measure of volatility persistence. Half-life gives the point estimate of the half-life in days given as $HL = \frac{\log(0.5)}{\log(\alpha+\beta)}$. `t-df` represents the Student's t degrees of freedom. GARCH, GJR and EGARCH are conditional heteroskedastic models defined in (4), (6) and (5), respectively

Table 13 GARCH, GJR and EGARCH(1,1), persistence and half-life—FACEBOOK

	Full sample	12/05/2014	28/04/2016	11/01/2018	20/02/2020	18/12/2020
ARCH-LM	20.50**	0.685	14.647	15.829	15.921	12.811
α	0.045*	0.006	0.156**	0.151***	0.112***	0.185*
β	0.950*	0.979*	0.051	0.386	0.624	0.681*
$\alpha + \beta$	0.996	0.985	0.207	0.537	0.736	0.867
Half-life	157.008	45.862	0.440	1.115	2.259	4.853
t-df	3.597*	4.338*	4.938*	4.070*	3.945	3.494*
γ GJR	0.054*	0.036	0.086	0.256**	0.209*	0.285*
γ EGARCH	– 0.043*	– 0.017	– 0.029	– 0.134**	– 0.121*	– 0.088*
BIC GARCH	4.123	5.020	3.957	3.074	3.083	4.346
BIC GJR	4.118	5.023	3.969	3.078	3.085	4.340
BIC EGARCH	4.110	5.033	3.971	3.073	3.079	4.339

ARCH-LM test. *, **, ***Denote statistically significant at the 1%, 5% and 10% significance levels, respectively. $\alpha + \beta = 1$ is the measure of volatility persistence. Half-life gives the point estimate of the half-life in days given as $HL = \frac{\log(0.5)}{\log(\alpha+\beta)}$. `t-df` represents the Student's t degrees of freedom. GARCH, GJR and EGARCH are conditional heteroskedastic models defined in (4), (6) and (5), respectively

cancel the effects of shocks on volatility. Due to the COVID-19 pandemic, persistence increased in 2020, but the estimates remain far from 1 in the case of FATANG stocks. If we go back to the period before COVID-19, only Google behaved in a similar fashion when compared to the US indices. Thus, we can conclude that bad news about FATANG stocks seems to have a shorter impact on volatility.

Persistence for the whole the sample is also substantially higher when compared to the persistence for each sub-sample; thus, splitting the sample according to the structural breaks can reduce the overall persistence. For example, in the case of Facebook, the estimated half-life[9] of the volatility persistence decreases

[9] Half-life gives the point estimate of the half-life in days given as $HL = \frac{\log(0.5)}{\log(\alpha+\beta)}$.

Table 14 GARCH, GJR and EGARCH(1,1), persistence and half-life—AMAZON

	Full sample	06/08/2002	24/04/2007	26/10/2009	10/05/2016	26/10/2017	18/12/2020
ARCH-LM	462.687*	21.554**	0.427	20.070***	9.216	38.621*	124.756*
α	0.038*	0.135*	−0.001	0.116*	0.047**	0.098***	0.264*
β	0.962*	0.732*	0.996	0.83*	0.801*	0.811*	0.709
$\alpha + \beta$	1.000	0.867	0.995	0.946	0.848	0.909	0.973
Half-life	7219.937	4.857	138.283	12.486	4.204	7.236	25.102
t-df	3.840*	5.143*	3.872*	3.739*	3.991*	6.135*	5.061*
γ GJR	0.036*	0.111***	0.001	0.230*	0.119*	0.203**	0.150**
γ EGARCH	−0.029*	−0.046	−0.002	−10.151*	−0.107*	−0.169*	−0.067***
BIC GARCH	4.756	6.434	4.514	5.251	4.146	3.132	4.051
BIC GJR	4.754	6.436	4.527	5.233	4.135	3.132	4.059
BIC EGARCH	4.736	6.431	4.517	5.223	4.129	3.129	4.055

ARCH-LM test. *, **, ***Denote statistically significant at the 1%, 5% and 10% significance levels, respectively. $\alpha + \beta = 1$ is the measure of volatility persistence. Half-life gives the point estimate of the half-life in days given as $HL = \frac{\log(0.5)}{\log(\alpha+\beta)}$. t-df represents the Student's t degrees of freedom. GARCH, GJR and EGARCH are conditional heteroskedastic models defined in (4), (6) and (5), respectively

Table 15 GARCH, GJR and EGARCH(1,1), persistence and half-life—TESLA

	Full sample	08/05/2014	26/03/2018	28/01/2020	18/12/2020
ARCH-LM	165.073*	34.563*	16.895	12.521	23.59**
α	0.041*	0.134**	0.111	0.086	0.160**
β	0.948*	0.553*	0.662**	0.694**	0.784*
$\alpha + \beta$	0.989	0.688	0.673	0.780	0.945
Half-life	60.701	1.850	1.753	2.786	12.145
t-df	3.542*	3.802*	4.475*	3.367*	5.908***
γ GJR	−0.004	−10.010	0.127*	0.186	0.005
γ EGARCH	−0.008	0.008	−0.189*	−10.088	−0.038
BIC GARCH	5.076	5.294	4.571	5.250	6.373
BIC GJR	5.080	5.300	4.563	5.259	6.3967
BIC EGARCH	5.077	5.297	4.571	5.257	6.393

ARCH-LM test. *, **, ***Denote statistically significant at the 1%, 5% and 10% significance levels, respectively. $\alpha + \beta = 1$ is the measure of volatility persistence. Half-life gives the point estimate of the half-life in days given as $HL = \frac{\log(0.5)}{\log(\alpha+\beta)}$. t-df represents the Student's t degrees of freedom. GARCH, GJR and EGARCH are conditional heteroskedastic models defined in (4), (6) and (5), respectively

from 157 days (when the whole sample is considered) to approximately to 2 and 5 days in the most recent sub-samples, which implies that a shock is expected to lose half of its original impact in just two or five days after the structural breaks are considered (see Table 13). According to this stylized fact of the returns' volatility, FATANG stocks are also substantially different from the US indices. After COVID-19 the estimated half-life in days ranges between 4.853 (Facebook) and 25.652 (Apple), while the estimate for US indices ranges between 76.704 (S&P 500) and 1571.224 (NASDAQ, where the GARCH(1,1) process is highly persistent, almost-integrated).

Finally, what conclusion can be drawn about the asymmetric effect on volatility? Despite taking into consideration the financial asset and the sub-sample,

Table 16 GARCH, GJR and EGARCH(1,1), persistence and half-life—APPLE

	Full sample	30/06/1997	03/10/2000	26/05/2009	20/02/2020	18/12/2020
ARCH-LM	42.814*	158.408*	9.040	97.877*	60.108*	39.313*
α	0.061*	0.036*	0.198*	0.029*	0.101*	0.139**
β	0.938*	0.893*	0.543*	0.961*	0.846*	0.834*
$\alpha + \beta$	0.999	0.929	0.741	0.990	0.947	0.973
Half-life	519.644	9.412	2.312	68.968	12.761	25.652
t-df	4.597*	4.041*	5.256	5.801*	4.463*	4.525*
γ GJR	0.039*	0.083*	0.015	0.040*	0.208*	0.162
γ EGARCH	− 0.033*	− 0.061*	0.015	− 0.045*	− 0.138*	− 0.115
BIC GARCH	4.533	4.809	5.620	4.867	3.656	4.947
BIC GJR	4.533	4.807	5.628	4.866	3.636	4.960
BIC EGARCH	4.521	4.805	5.634	4.864	3.628	4.962

ARCH-LM test. *, **, ***Denote statistically significant at the 1%, 5% and 10% significance levels, respectively. $\alpha + \beta = 1$ is the measure of volatility persistence. Half-life gives the point estimate of the half-life in days given as $HL = \frac{\log(0.5)}{\log(\alpha+\beta)}$. `t-df` represents the Student's t degrees of freedom. GARCH, GJR and EGARCH are conditional heteroskedastic models defined in (4), (6) and (5), respectively

Table 17 GARCH, GJR and EGARCH(1,1), persistence and half-life—NETFLIX

	Full sample	18/10/2004	14/09/2011	23/04/2013	18/12/2020
ARCH-LM	32.096*	11.149	8.961	1.430	9.864
α	0.031*	0.235**	0.089*	0.177	0.110*
β	0.963*	0.377***	0.761*	0.251	0.822*
$\alpha + \beta$	0.994	0.612	0.850	0.428	0.931
Half-life	109.711	1.412	4.265	0.817	9.729
t-df	3.175*	3.217*	3.526*	2.710*	3.357*
γ GJR	0.018**	0.396***	0.051	− 0.131	0.153*
γ EGARCH	− 0.029*	− 0.169**	− 0.021	0.032	− 0.072*
SIC GARCH	4.995	5.975	5.011	5.635	4.570
SIC GJR	4.996	5.979	5.014	5.643	4.564
SIC EGARCH	4.975	5.985	5.007	5.638	4.549

ARCH-LM test. *, **, ***Denote statistically significant at the 1%, 5% and 10% significance levels, respectively. $\alpha + \beta = 1$ is the measure of volatility persistence. Half-life gives the point estimate of the half-life in days given as $HL = \frac{\log(0.5)}{\log(\alpha+\beta)}$. `t-df` represents the Student's t degrees of freedom. GARCH, GJR and EGARCH are conditional heteroskedastic models defined in (4), (6) and (5), respectively

the asymmetric GJR and EGARCH models almost completely dominate the symmetric GARCH model, which means that negative shocks (when compared to the positive ones) have a stronger impact on returns volatility. The estimates for the asymmetric coefficient γ have the correct sign (positive in the case of GJR and negative in the case of EGARCH), and they are statistically significant for most of the sub-samples. The Bayesian information criterion (BIC) also favors the asymmetric conditional heteroskedasticity models. There are just a few exceptions where positive and negative shocks have the same impact on volatility and the symmetric GARCH beats the other two models. In the most recent sub-sample, after COVID-19, the estimate for the coefficient of asymmetry (γ) is not statistically significant in the cases of the three US indices: S&P

Table 18 GARCH, GJR and EGARCH(1,1), persistence and half-life—GOOGLE

	Full sample	12/05/2006	07/11/2007	23/01/2009	20/02/2020	18/12/2020
ARCH-LM	66.915*	27.295*	5.704	9.410	17.472	34.527*
α	0.063*	0.090**	0.036	0.081	0.030*	0.161**
β	0.927*	0.838*	0.471	0.873*	0.954*	0.797*
$\alpha+\beta$	0.990	0.928	0.507	0.954	0.984	0.959
Half-life	66.308	9.276	1.02	14.719	44.042	16.422
t-df	3.889*	4.469*	5.471*	4.149*	3.834*	4.872**
γ GJR	0.071*	0.032	– 0.046	0.156***	0.058*	0.258**
γ EGARCH	– 0.052*	– 0.019	– 0.016	– 0.254*	– 0.065*	– 0.120***
BIC GARCH	3.756	4.593	3.667	5.183	3.459	4.577
BIC GJR	3.751	4.607	3.682	5.158	3.453	4.579
BIC EGARCH	3.742	4.611	3.685	5.204	3.444	4.585

ARCH-LM test. *, **, ***Denote statistically significant at the 1%, 5% and 10% significance levels, respectively. $\alpha+\beta=1$ is the measure of volatility persistence. Half-life gives the point estimate of the half-life in days given as $HL=\frac{\log(0.5)}{\log(\alpha+\beta)}$. `t-df` represents the Student's t degrees of freedom. GARCH, GJR and EGARCH are conditional heteroskedastic models defined in (4), (6) and (5), respectively

Table 19 Volatility persistence in the sub-samples before and after COVID-19

	Before COVID-19		After COVID-19	
Indices/Stocks	$\alpha+\beta$	Half-life	$\alpha+\beta$	Half-life
S&P 500	0.965	19.654	0.991	76.704
DJIA	0.984	42.418	0.997	272.224
NASDAQ	0.977	29.802	1.000	1571.416
FACEBOOK	0.736	2.259	0.867	4.853
AMAZON	0.909	7.236	0.973	25.102
TESLA	0.780	2.786	0.945	12.145
APPLE	0.947	12.761	0.973	25.652
NETFLIX	0.428	0.817	0.931	9.729
GOOGLE	0.984	44.042	0.959	16.422

500, NASDAQ (EGARCH estimate), DJIA plus Tesla and Apple. Thus, it would appear that bad news still has a higher (even shorter) impact on volatility in the case of FATANG stocks; the difference between the impact of positive and negative news on volatility seems lower in the case of US indices.

6 Conclusions and future directions for research

Volatility of financial asset returns has been extensively studied for the last thirty years, and it remains a very important topic of investigation due to its importance for investors, financial analysts and academics. In this paper, we analyze how the volatility of returns of three US stock indices: S&P500, DJIA and NASDAQ, and six US stocks under the acronym FATANG: Facebook, Amazon, Tesla, Apple, Netflix and Google, representing the new economy, have evolved over recent decades.

First we analyzed the dispersion, skewness and kurtosis of the empirical distributions of returns. To notice the temporal changes, all the measures were computed over a rolling window encompassing the previous year of daily observations ($T=250$). Then, based on the unconditional standard deviation, we observed a sharp decrease in volatility after the year 2012, regardless of the index or the stock being considered. For Amazon and Apple, for example, the variance of returns before 2012 was almost 6 and 4 times higher than the variance computed for the following six years. In terms of skewness and kurtosis, when the stock indices were considered, most of the estimates of skewness were negative, with just a few being statistically significant and positive. Furthermore, the peaks in the kurtosis were mostly due to large negative returns, giving rise to the negative spikes of the coefficient of skewness. The conclusions are different when the individual FATANG stocks were analyzed. The estimates of skewness could be either positive or negative, and the peaks of kurtosis were also explained by large positive returns leading to the positive spikes of the coefficient of skewness. Thus, large positive returns also inflate the kurtosis. By separating the "bad" volatility and kurtosis from the "good", because the former represents risk while the latter only represents uncertainty, and based on semi-variance and semi-kurtosis measures, we concluded that while the volatility and kurtosis of FATANG stocks returns represent more uncertainty than risk, they represent more risk than uncertainty in the case of stock indices. Thus, the indices would appear to be riskier than that particular class of tech stocks.

Second, we analyzed the autocorrelation of returns, absolute returns and squared returns. According to the Ljung–Box test for returns, the autocorrelation seems not to be relevant (only in a small percentage of windows is the "no autocorrelation" null hypothesis rejected, with the exception being the NASDAQ composite index with more than 20% of rejections). The autocorrelation structure of returns still seems weaker in the case of individual stocks. (In the case of Tesla, for example, the number of rejections is almost zero.) Even though the series of returns seemed to be weakly correlated over time, the autocorrelation of absolute and squared returns was stronger, pointing to a positive autocorrelation over several days, which quantifies the fact that high volatility events tend to cluster in time. However, the volatility clustering stylized fact is not so evident for individual stocks, especially in the cases of Amazon, Tesla and Netflix. The empirical results seem also to confirm that sample autocorrelations for absolute returns are greater than the sample autocorrelations for squared returns. Since one possible explanation for the large positive autocorrelation between $|r_t|$ and r_t^2 is the heteroskedasticity of returns, i.e., the variance or conditional variance changes over time, we also computed the ARCH-LM test. The results suggest that all financial asset returns exhibit ARCH effects and conditional heteroskedasticity. However, this empirical result is not so evident in the case of FATANG stocks.

Finally, we modeled time-varying conditional volatility. As several studies show that structural breaks have potential implications regarding the estimation results, we first tested for the existence of structural breaks in volatility of the twelve returns series by applying a modified version of the iterated cumulative sum of squares (ICSS) algorithm that allows for temporal dependence. The results show strong evidence of structural breaks in the unconditional variance for all the returns series, leading to distinct regimes in volatility. Next we estimated an ARMA(4,0) model for the conditional mean in combination with GARCH(1,1), GJR(1,1) and EGARCH(1,1) models for the conditional variance across the sub-samples resulting from the structural breaks identified by the modified ICSS algorithm.

Three main conclusions can be drawn. First, the ARCH-LM test points for the existence of conditional heteroskedasticity in most of the series and for most of the sub-samples resulting from the structural breaks. However, in the most recent past there have been four exceptions: Facebook, Tesla, Netflix and Google. Thus, the suspicion of constant variance of returns is more evident among the FATANG stocks. Second, GARCH(1,1) processes are highly persistent (almost-integrated) when estimated over the full sample, with the estimate for $\alpha + \beta$ ranging between 0.989 and 1. If we focus mainly on the sub-sample resulting from the last structural break, the estimates range between 0.867 (Facebook) and 1 (NASDAQ). The decrease in volatility persistence is also a symptom of less volatility in the stock markets. Finally, the asymmetric GJR and EGARCH models almost completely dominate, despite taking into consideration the financial asset and the sub-sample being considered, the symmetric GARCH model, which means that negative shocks (when compared to positive ones) have a stronger impact on returns volatility.

This paper opens up at least three new lines of future investigation. Firstly, the newly proposed downside risk measure can be assessed (and compared to traditional measures) in different classes of financial assets, namely interest rates and exchange rates (including crypto currencies). Secondly, due to the role of tech stocks in the US capital markets, it is important to test whether there is a volatility spillover effect and whether it runs one-way from FATANG stocks to US stock indices, or whether there is a feedback effect. Finally, major empirical findings point to volatility decreasing in the most recent past, excluding March 2020. Thus, should we keep faith with homoskedasticity or trust that a new heteroskedasticity regime will come soon? Is the 2020 increase in volatility evidence of on-going structural change, or is it just a scare for investors? Further empirical research is needed, therefore, to shed light on volatility and to provide investors with guidance as to risk in the capital markets in general, and in FATANG stocks in particular.

Acknowledgements I would like to thank the Editor-in-Chief Walter Lacarbonara, the Associate Editor J. A. Tenreiro Machado and two anonymous referees for their suggestions and comments.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Availability of data and material The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

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